Robust Cooperative Navigation for AUVs using the Student's t Distribution

Qian Li[1],Syed Mohsen Naqvi[2], Jeff Neasham[2], Jonathon Chambers[2]

1. School of Electrical Engineering and Automation, Harbin Institute of Technology, China
2. Intelligent Sensing and Communications Group, School of Engineering, Newcastle University, UK

Email: liqianheu@163.com, {Mohsen.Naqvi, Jeff.Neasham, Jonathon.Chambers}@ncl.ac.uk

Abstract

In a cooperative navigation system for multiple Autonomous Underwater Vehicles (AUVs), an acoustic communication technique is usually used to exchange information and measure range between AUVs, and many cheap but low-accuracy Micro-Electro-Mechanical Systems (MEMS)-based Inertial Measurement Units (IMUs) are used as Dead-Reckoning (DR) sensors on AUVs. The use of unreliable sensors and an acoustic communication technique can induce outliers leading to the probability densities of process and measurement noise having a heavier-tailed behavior than a Gaussian distribution. To cope with such non-Gaussian distributions, the process and measurement noises are modeled as Student's t distributions, and the Student's t filtering algorithm for cooperative navigation is presented. Simulation results show the efficiency and superiority of the proposed robust cooperative navigation algorithm as compared with the standard extended Kalman filtering-based cooperative navigation algorithm.

Introduction

Accurate navigation is a vital enabler for the operation of an Autonomous Underwater Vehicle (AUV) and it is also essential to improve the efficiency of AUV missions. Aiming at the deployment of multiple AUVs, Cooperative Navigation (CN) is a viable option for high accuracy underwater navigation of multiple AUVs. In CN, a fleet of AUVs exchange relative position measurements from their exteroceptive sensors and their motion information from proprioceptive sensors to collectively estimate their states. The study indicates that:

- The exchange of positioning information benefits all vehicles.
- If absolute geo reference information could be provided to one of the vehicles, the states of vehicles performing CN are observable in a connected Relative Position Measurement Graph (RPMG).

Such increase in navigation accuracy is a major benefit to CN, and its advantages also include sensor coverage, robustness and flexibility, and thus it remains an active area of research.

Many CN algorithms which could make a consistent and accurate estimation of the positions of the full fleet of vehicles have been proposed. However, most approaches proposed assume that the process and measurement noises admit a Gaussian distribution. In fact, the Gaussian distribution assumption of the process and measurement noises is usually violated by some outliers induced by:

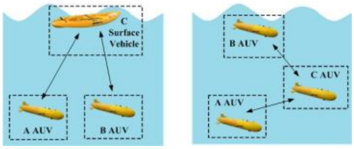
- Low-accuracy and unreliable DR sensors present on slave AUVs, such as a Micro-Electro-Mechanical Systems (MEMS)-based Inertial Measurement Unit (IMU).
- Underwater acoustic communication system.

Thus, the process and measurement noise contaminated by the outliers from unreliable MEMS-based IMU and the underwater acoustic range measurements are modeled by a Student's t distribution in this paper, and a new robust CN algorithm for multiple AUVs based on a Student's t distribution is proposed.

System Model and Noise Analysis

A typical framework of master-slave CN for three AUVs is considered in this paper. In this configuration, A and B are two slave AUVs, which are both equipped with low-cost and low-accuracy compass and speed sensor. To bound the navigation error of the slave AUVs, a master AUV (C AUV) with an expensive and accurate navigation suite is included in the CN system. With the information about ranges to the master AUV and the accurate position of master AUV, the unbounded navigation errors of the slave AUVs are corrected by a filter technique. The discrete-time kinematic equations on the x-y horizontal plane for the i-th AUV of a fleet of AUVs are:

$$\begin{cases} x_{k+1}^i = x_k^i + t\cdot V_k^i \cdot \cos(\theta_k^i) \\ y_{k+1}^i = y_k^i + t\cdot V_k^i \cdot \sin(\theta_k^i) \\ \theta_{k+1}^i = \theta_k^i + t\cdot \omega_k^i \end{cases}$$



Considering the discrete-time kinematic equations for the i-th AUV, the CN dynamic model is as below:

$$X_{k+1} = f(X_k, u_k, w_k) = X_k + t\cdot G_k(u_k + w_k)$$

The range between A AUV and C AUV is calculated as below:

$$z_k^{C\to A} = \sqrt{(x_k^C - x_k^A)^2 + (y_k^C - y_k^A)^2} + v_k$$

The measurement equation is nonlinear and the linearized measurement matrix is represented by:

$$H_k^{C\leftrightarrow A} = \left[\frac{\hat{x}_k^A - \hat{x}_k^C}{r_{AC}} \quad \frac{\hat{y}_k^A - \hat{y}_k^C}{r_{AC}} \quad 0_{1\times 4} \quad \frac{\hat{x}_k^C - \hat{x}_k^A}{r_{AC}} \quad \frac{\hat{y}_k^C - \hat{y}_k^A}{r_{AC}} \quad 0\right]$$

To cope with the outliers in the process and measurement noise, we introduce the multivariate Student's t distribution to describe the process and measurement noises as below:

$$St(X;\hat{X},P,\upsilon) = \frac{\Gamma\left(\frac{\upsilon+n}{2}\right)}{\Gamma\left(\frac{\upsilon}{2}\right)} \frac{1}{(\upsilon\pi)^{\frac{n}{2}}} \frac{1}{\sqrt{\det(P)}}$$

$$\left(1+\frac{1}{\upsilon}\left(X-\hat{X}\right)^T \; P^{-1}\left(X-\hat{X}\right)\right)^{-\frac{n+\upsilon}{2}}$$

In Fig.1, the posterior probability density function (pdf) of several Student's t distributions with different dof values are drawn and compared.

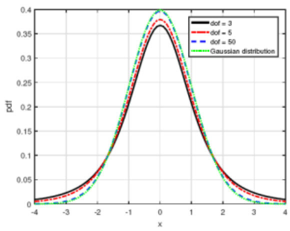


Fig. 1 Comparison of Gaussian distribution (green dot line) and Student's t distribution with different dof values.

Cooperative Navigation Algorithm based on Nonlinear Student's t Filter

We propose a robust cooperative navigation algorithm based on a Student's t filter. Roth et al. proposed a Student's t filter for linear systems by approximating the posterior pdf as a Student's t distribution [1]. However, for a CN system, the process and measurement equations are nonlinear. Thus, directly porting the linear Student's t filter algorithm to the cooperative navigation case is not straightforward. Fortunately, both the Gaussian distribution and the Student's t distribution are closed under linear transformation, thus the framework proposed by Roth et al. can be extended to nonlinear systems in a manner similar to the development of the EKF for Gaussian systems.

The details of the proposed nonlinear Student's t filter algorithm (NSTF) are summarized in the table NSTF Algorithm.

Nonlinear Student's t Filter (NSTF) Algorithm

Inputs: Initialize $\hat{X}_0$, P_0, Q_k, R_k, Q_k, $Z_{1:T}$, η_0, γ , δ

For $k = 1{:}T$ Perform the following:

a. Perform the approximation of common dof and adjustment of matrix parameters.

$$\eta_{k-1}' = \min(\eta_k, \gamma)\,, P_{k-1}^+ \to \tilde{P}_{k-1}^+$$

b. Linearize the process equation and perform the time update of the state estimate together with symmetric matrix.

$$\hat{X}_k^- = f_k(\hat{X}_{k-1}^+, u_{k-1}, 0)$$

$$P_k^- = F_{x_{k-1}} \tilde{P}_{k-1}^+ F_{x_{k-1}}^T + F_{u_{k-1}} Q_{k-1} F_{u_{k-1}}^T$$

c. Perform the approximation of common dof and adjustment of symmetric matrix, if $\gamma \neq \delta$.

d. Linearize the measurement equation and perform the measurement update of the state estimate and symmetric matrix.

$$\hat{X}_k^+ = \hat{X}_k^- + P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} (z_k - h_k(\hat{X}_k^-, 0))$$

$$\tilde{P}_k^+ = \frac{\eta_{k-1} + \Delta_2^2}{\eta_{k-1} + n_{dz}} (P_k^- - P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} H_k P_k^-)$$

e. Update the dof.

$$\eta_{k-1} \to \eta_k$$

Outputs: $\{\hat{X}_k^+, \hat{P}_k^+ \; 0 \le k \le T\}$

Simulation Results

The standard deviations of the range measurement and position measurement are $\sigma_r = 4m, \sigma_p = 4m$, respectively. The standard deviations of the velocity and rotational velocity of A and B AUV are set to be $\sigma_V = 1m/s, \sigma_\omega = 0.2\text{deg}/s$. The process noise and measurement noise are set to follow Student's t distributions, and the dof for the Student's t distributions were both chosen as 3

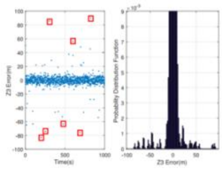


Fig. 2 Measurement noise and its pdf, and the dots marked by red squares are some outliers.

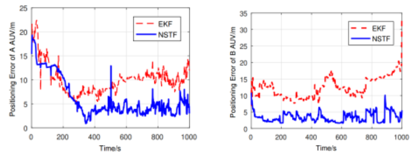


Fig. 3. Positioning error of A and B AUV in the CN system.

As it can be seen, the average positioning errors of A AUV and B AUV are 10.13m and 12.42m using EKF, respectively. Estimated by the NSTF, the average positioning errors are reduced to be 6.14m and 3.14m, and the positioning accuracy is improved by 39.4% and 74.7%, respectively.

Conclusion

It was found that the CN algorithm based on the Student's t distribution outperformed the standard EKF in terms of positioning error when the process and measurement noise had heavy-tailed behavior.

Selected references

[1] M. Roth, E. O¨ zkan, and F. Gustafsson, "A Student's t filter for heavy tailed process and measurement noise," in IEEE International Conference on Acoustics, Speech and Signal Processing, 2013, pp. 5770–5774.

Newcastle University