Impact of Fast-Converging PEVD Algorithms on Broadband AoA Estimation

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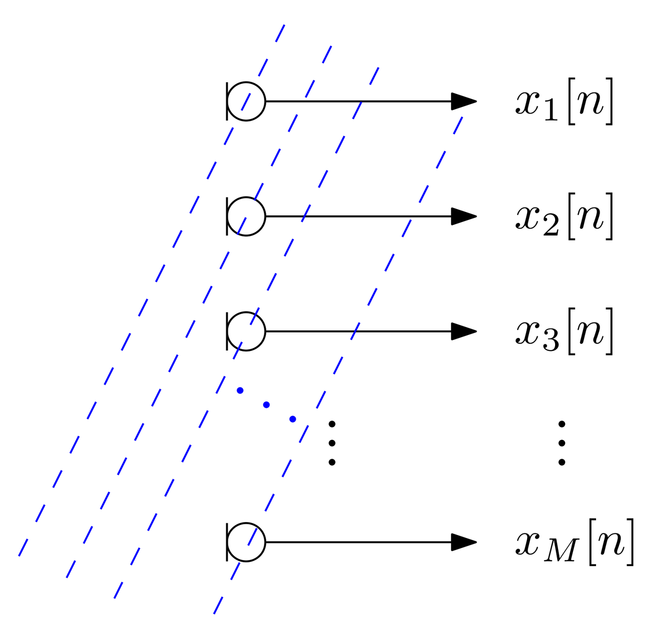
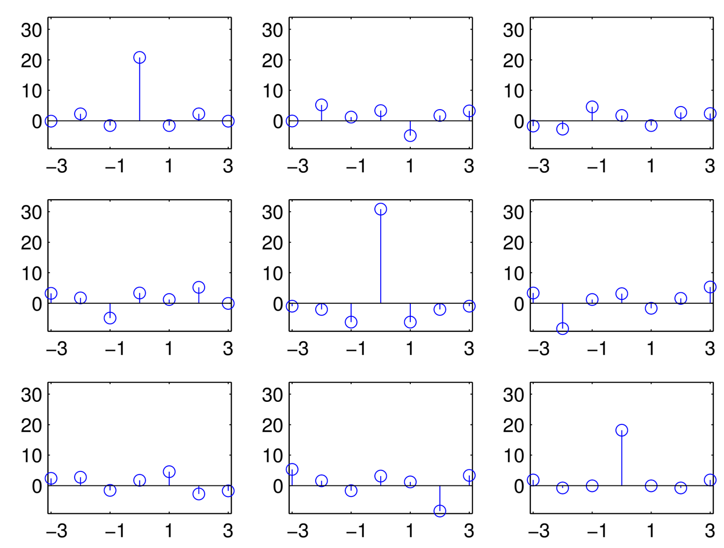
Background

Motivation: Polynomial matrix eigenvalue decomposition (PEVD) algorithms have been shown to enable a solution to the broadband angle of arrival (AoA) estimation problem.

Aim: Employ low complexity divide-and-conquer approach to the PEVD for AoA estimation and investigate performance relative to traditional PEVD methods. Simultaneously, quantify the performance trade-offs for divide-and-conquer algorithm parameters.



- Space-time covariance matrix: $\mathbf{R}[\tau] = \mathcal{E}\{\mathbf{x}[n]\mathbf{x}^{\mathrm{H}}[n-\tau]\}$, $\mathbf{R}[\tau] \in \mathbb{C}^{M\times M}$
- Matrix of auto- & cross- correlation sequences
- Symmetry: $\mathbf{R}[\tau] = \mathbf{R}^{\mathrm{H}}[-\tau]$



- Cross spectral density $\boldsymbol{R}(z) = \sum_\tau \mathbf{R}[\tau]z^{-\tau}$ is a polynomial matrix.
- Parahermitian: $\tilde{\boldsymbol{R}}(z) = \boldsymbol{R}^{\mathrm{H}}(1/z^*) = \boldsymbol{R}(z)$

Spatio-Spectral Polynomial MUSIC

- Approximate Polynomial EVD [1]: $\boldsymbol{D}(z) \approx \tilde{\boldsymbol{Q}}(z)\boldsymbol{R}(z)\boldsymbol{Q}(z)$
- Thresholding the polynomial eigenvalues reveals the number of independent broadband sources R contributing to $\boldsymbol{R}(z)$, and permits a distinction between signal-plus-noise and noise only subspaces $\boldsymbol{Q}_s(z) \in \mathbb{C}^{M\times R}$ and $\boldsymbol{Q}_n(z) \in \mathbb{C}^{M\times(M-R)}$,

$$\boldsymbol{R}(z) = \begin{bmatrix} \boldsymbol{Q}_s(z) & \boldsymbol{Q}_n(z) \end{bmatrix} \begin{bmatrix} \boldsymbol{D}_s(z) & \mathbf{0} \\ \mathbf{0} & \boldsymbol{D}_n(z) \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{Q}}_s(z) \\ \tilde{\boldsymbol{Q}}_n(z) \end{bmatrix} .$$

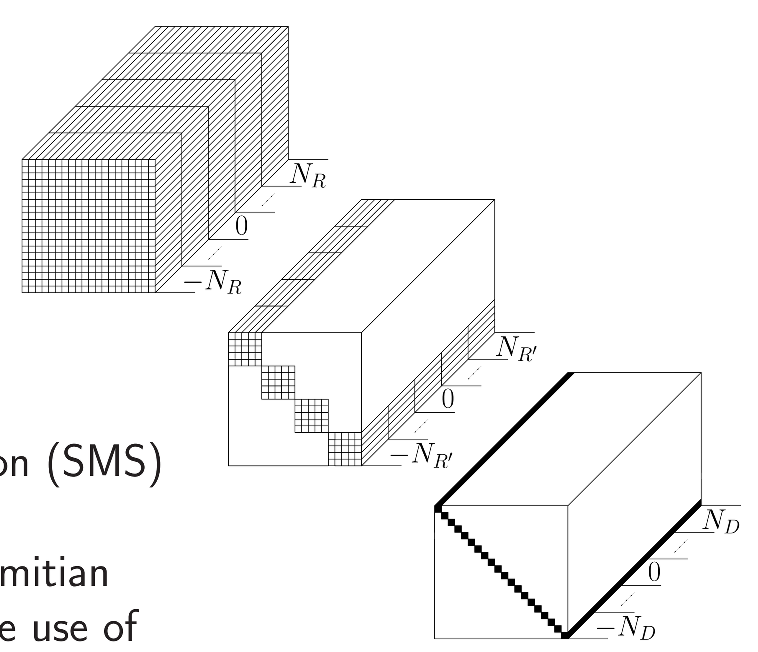
- $R < M$, $\boldsymbol{D}_s(z) \in \mathbb{C}^{R\times R}$ and $\boldsymbol{D}_n(z) \in \mathbb{C}^{(M-R)\times(M-R)}$.
- The spatio-spectral polynomial MUSIC (SSP-MUSIC) algorithm [2] is an extension of narrowband MUSIC [3] to the broadband case.
- SSP-MUSIC algorithm scans the noise-only subspace $\boldsymbol{Q}_n(z) = [\boldsymbol{Q}_{R+1}(z) \ \ldots \ \boldsymbol{Q}_M(z)]$.
- The steering vectors of sources that contribute to $\boldsymbol{R}(z)$ will define the signal-plus-noise subspace $\boldsymbol{Q}_s(z)$ and therefore lie in the nullspace of its complement $\boldsymbol{Q}_n(z)$.
- Vector $\tilde{\boldsymbol{Q}}_n(e^{j\Omega})\boldsymbol{A}_{\vartheta,\varphi}(e^{j\Omega})$ is close to the origin if $\boldsymbol{A}_{\vartheta,\varphi}(e^{j\Omega})$ is a steering vector of a contributing source at frequency Ω, azimuth φ, and elevation θ.
- The SSP-MUSIC algorithm evaluates the reciprocal of the norm of this vector,

$$P_{\mathrm{SSP}}(\vartheta,\varphi,e^{j\Omega}) = \frac{1}{\tilde{\boldsymbol{A}}_{\vartheta,\varphi}(z)\boldsymbol{Q}_n(z)\tilde{\boldsymbol{Q}}_n(z)\boldsymbol{A}_{\vartheta,\varphi}(z)}\Big|_{z=e^{j\Omega}} .$$

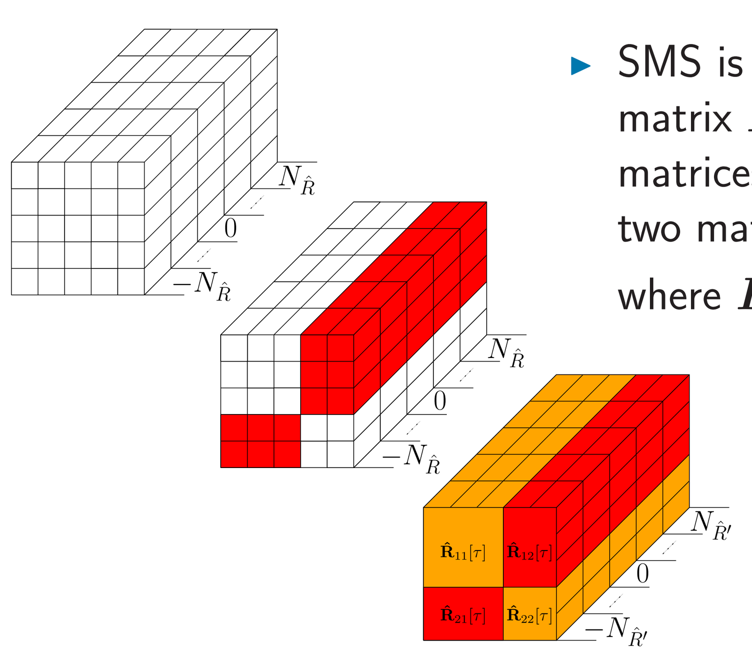
- $P_{\mathrm{SSP}}(\vartheta,\varphi,e^{j\Omega})$ can determine over which frequency range sources in the direction defined by the steering vector $\boldsymbol{A}_{\vartheta,\varphi}(z)$ are active.

Divide-and-Conquer Sequential Matrix Diagonalisation

- Work in [4] describes a divide-and-conquer approach for the PEVD. This algorithm — titled divide-and-conquer sequential matrix diagonalisation (DC-SMD) — can be utilised to reduce algorithm complexity and has a framework based on the SMD [5] algorithm.
- While traditional PEVD algorithms attempt to diagonalise an entire $M\times M$ parahermitian matrix at once, the DC-SMD algorithm first divides the matrix into a number of smaller, independent parahermitian matrices, before diagonalising — or conquering — each matrix separately.



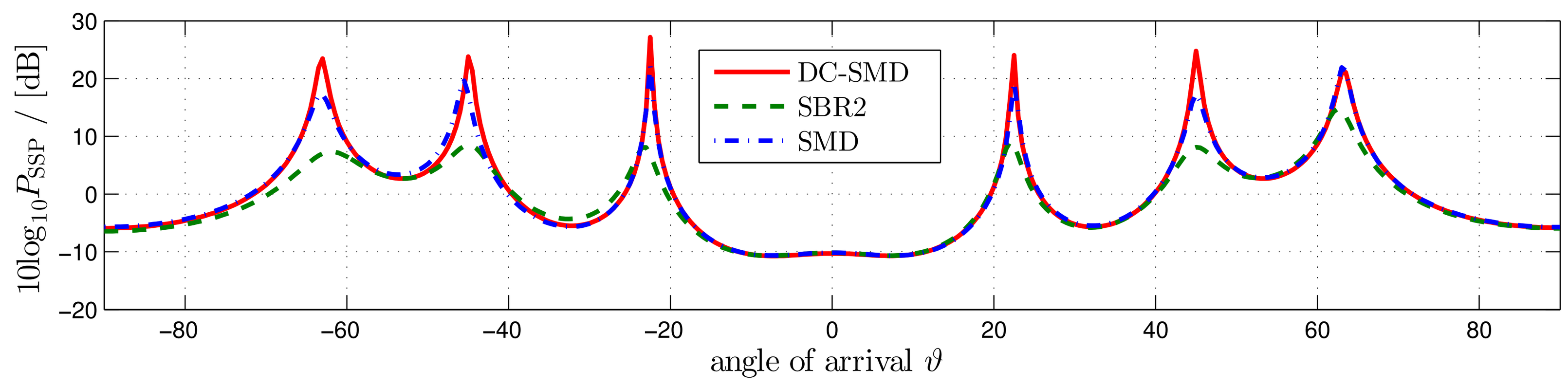
- An algorithm named sequential matrix segmentation (SMS) [4] is used to recursively divide $\boldsymbol{R}(z)$ into multiple independent parahermitian matrices. Each parahermitian matrix is then diagonalised in sequence through the use of the SMD algorithm.

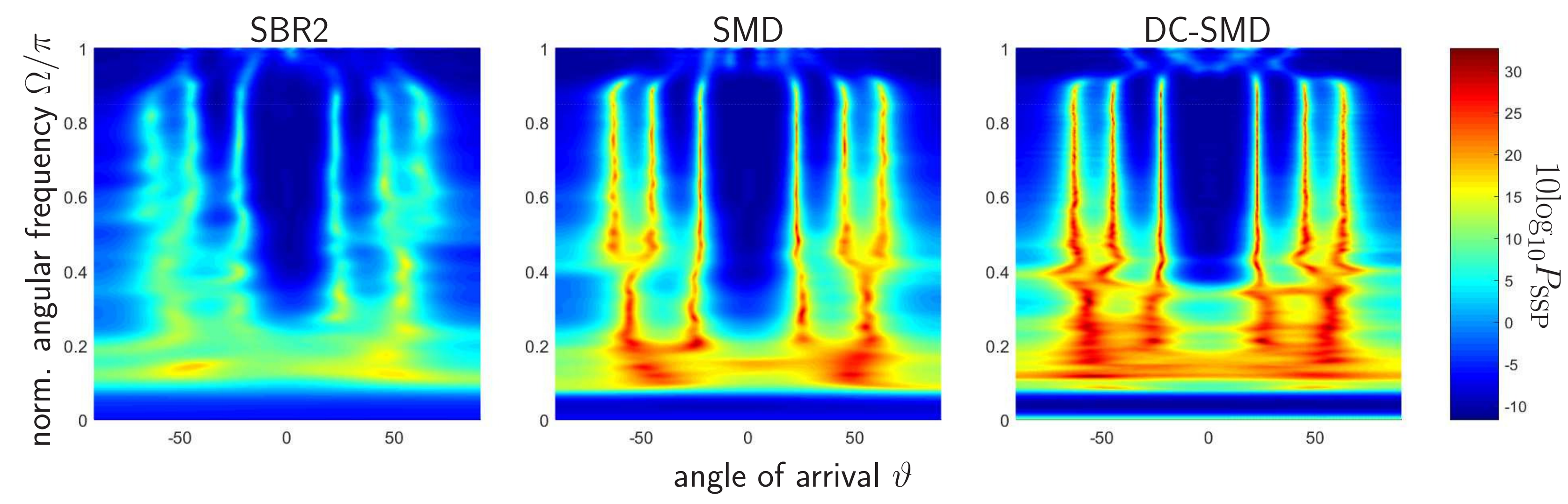


- SMS is a novel variant of SMD designed to segment an input matrix $\hat{\boldsymbol{R}}(z) \in \mathbb{C}^{M'\times M'}$ into two independent parahermitian matrices $\hat{\boldsymbol{R}}_{11}(z) \in \mathbb{C}^{(M'-P)\times(M'-P)}$ and $\hat{\boldsymbol{R}}_{22}(z) \in \mathbb{C}^{P\times P}$, and two matrices $\hat{\boldsymbol{R}}_{12}(z) \in \mathbb{C}^{(M'-P)\times P}$ and $\hat{\boldsymbol{R}}_{21}(z) \in \mathbb{C}^{P\times(M'-P)}$, where $\hat{\boldsymbol{R}}_{12}(z) = \tilde{\hat{\boldsymbol{R}}}_{21}(z)$ are approximately zero.
- SMS iteratively minimises the energy in select regions of a parahermitian matrix in an attempt to segment the matrix. SMS operates until a specified number of iterations have been executed, or when the energy in the targeted regions falls below a threshold.

Results

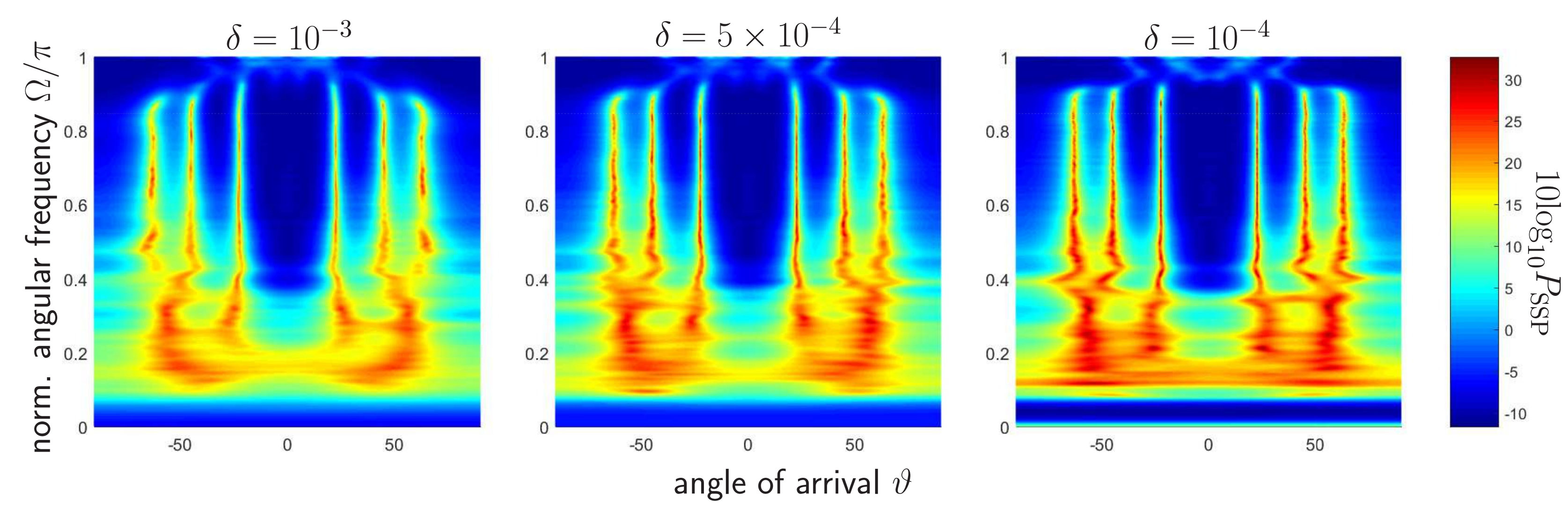
- Impact example: broadband angle of arrival estimation with fixed execution time.

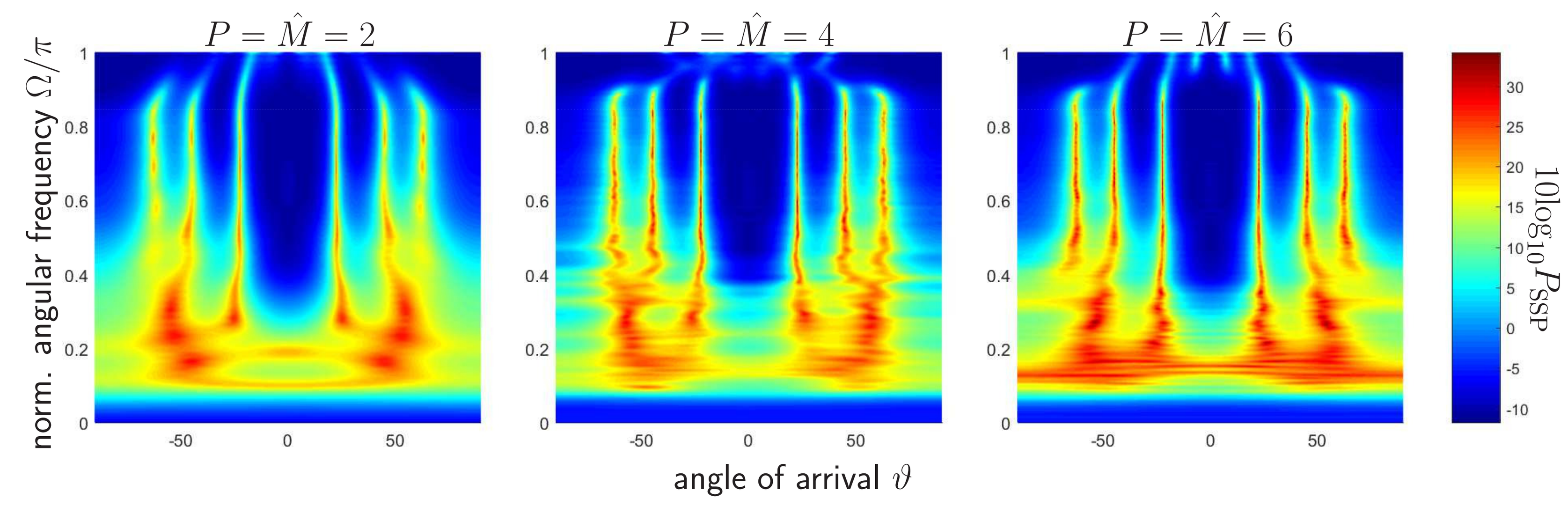


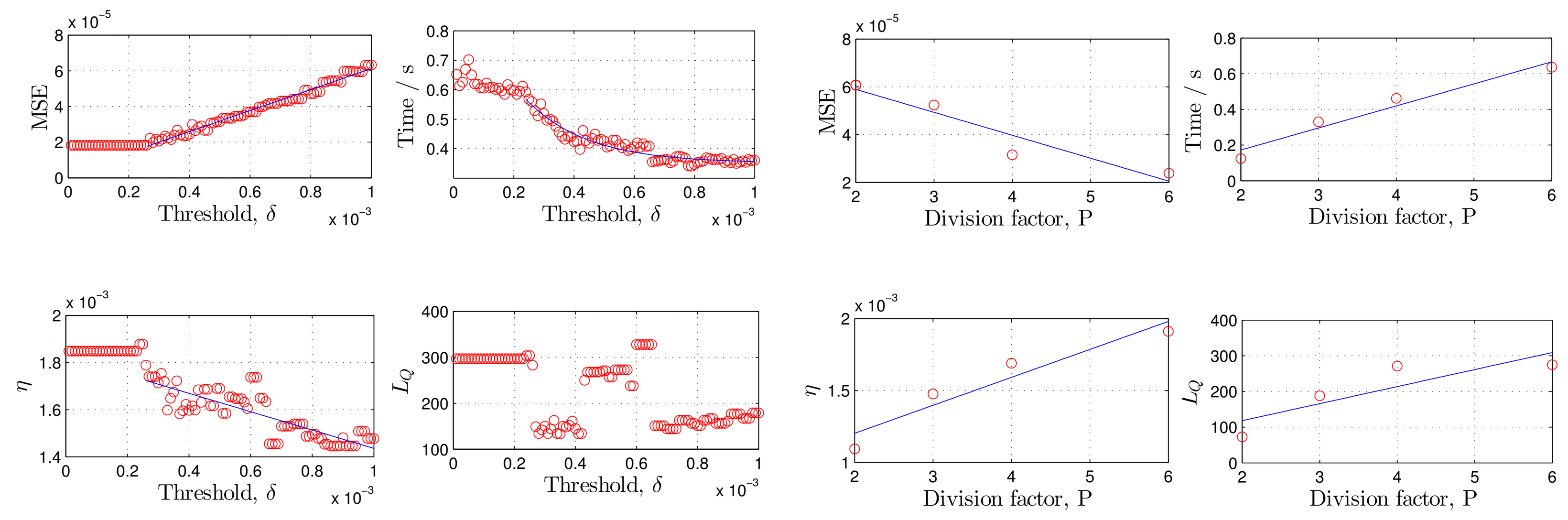


Algorithm	Diagonalisation / dB	Paraunitary filter length	Decomposition error	Paraunitarity error
DC-SMD	-17.17	297	1.27×10^{-5}	1.83×10^{-3}
SBR2	-8.035	221	7.19×10^{-9}	4.91×10^{-5}
SMD	-12.87	170	1.15×10^{-7}	1.94×10^{-4}

- Performance trade-offs of DC-SMD for fixed diagonalisation level.







Conclusions

- DC-SMD offers significant AoA estimation and diagonalisation performance gains over traditional PEVD algorithms for equal execution time.
- These benefits come with the disadvantage of increasing the mean squared reconstruction error, paraunitary filter length, and paraunitarity error.
- Through careful choice of DC-SMD input parameters δ, P, and $\hat{M}$, a balance can be obtained between decomposition MSE, algorithm execution time, filter paraunitarity, paraunitary filter length, and AoA estimation performance.
- A further advantage of the DC-SMD algorithm is its ability to produce multiple independent parahermitian matrices, which may be processed in parallel.

References

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