Multi-sensor multi-target tracking techniques for Space Situational Awareness

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China's Tiangong-1 space station 'out of control' and will crash to Earth

Chinese authorities confirm the eight-tonne ‘Heavenly Palace’ lab will re-enter the atmosphere sometime in 2017 with some parts likely to hit Earth.

-China's Long March 2-F rocket, which took the Tiangong-1 space module into space. Photograph: STR/AFP/Getty Images
Multi-sensor multi-target tracking techniques for Space Situational Awareness

**Motivation:** Methods for tracking space debris are essential to prevent damage to expensive space-related infrastructure and to determine cause.

- **Examples of recent events:**
  - 2009 Russian Kosmos 2251/US Iridium 33 collision.

**Objective:** Develop methods for estimation of populations of objects in orbit from sensor data.


Multi-sensor multi-target tracking techniques for Space Situational Awareness

Topics:
1. Tracking trajectories of individual objects
2. Multi-target tracking
3. Joint sensor motion, target tracking, and classification
Markov transition density

\[ p_{k+1|k}(x_{k+1} | z_{1:k}) = \int f_{k+1|k}(x_{k+1} | x)p_k(x | z_{1:k}) dx \]
TARGET TRACKING: UPDATE

\[ p_{k+1}(x_{k+1}|z_{1:k}) = \frac{g_{k+1}(z_{k+1}|x_{k+1})p_{k+1|k}(x_{k+1}|z_{1:k})}{\int g_{k+1}(z_{k+1}|x)p_{k+1|k}(x|z_{1:k})dx} \]

Conditional likelihood
TARGET TRACKING: ORBITING OBJECTS
TRACKING A SATELLITE FROM LASER RANGING

Particle Filter

- Data
- Prediction
- Weighted average of particles
- Highest/Lowest particle

Range (metres)

Time (seconds past midnight)
**Chilbolton Advanced Meteorological Radar**

- Fully steerable meteorological 3Ghz radar with a Doppler capability
- Modified in 2010 to carry out Space Situational Awareness (SSA) operations
- Low Earth Orbit (LEO) object tracking

Image Credit: [http://www.metoffice.gov.uk/](http://www.metoffice.gov.uk/)
MULTI-OBJECT FILTERING

observation set produced by targets

observation space

state space

target motion

state-set

5 targets

3 targets

multi-object Bayes filter

\[ \ldots \rightarrow p_{k-1}(X_{k-1} | Z_{1:k-1}) \xrightarrow{\text{prediction}} p_{k|k-1}(X_k | Z_{1:k-1}) \xrightarrow{\text{data-update}} p_k(X_k | Z_{1:k}) \rightarrow \ldots \]

\[ \int f_{k|k-1}(X_k | X_{k-1}) p_{k-1}(X_{k-1} | Z_{1:k-1}) \, \delta X_{k-1} \]

Markov Transition

\[ K^{-1} p_{k|k-1}(X_k | Z_{1:k-1}) g_k(Z_k | X_k) \]

Likelihood

new observation
TRACKING MULTIPLE ORBITING OBJECTS
A spatial point process is a probabilistic representation of a random set of objects
For example:

- 2-dimensional positions of objects in an image from a sensor (i.e. an observation space)

- 3-dimensional positions and velocities of objects in some real-world environment (i.e. a state space).
Point processes

<table>
<thead>
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<th>Number of objects</th>
<th>Cardinality probability</th>
<th>Joint spatial density</th>
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<td>-</td>
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<tr>
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<td>$\rho(1)$</td>
<td>$p_1(x_1)$</td>
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<td>$\rho(2)$</td>
<td>$p_2(x_1, x_2)$</td>
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<td>$p_3(x_1, x_2, x_3)$</td>
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<tr>
<td>4</td>
<td>$\rho(4)$</td>
<td>$p_2(x_1, x_2, x_3, x_4)$</td>
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<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>n</td>
<td>$\rho(n)$</td>
<td>$p_n(x_1, x_2, x_3, x_4, \ldots, x_n)$</td>
</tr>
<tr>
<td>...</td>
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</tbody>
</table>

Representation: The probability generating functional (p.g.fl.)

$$G_\Phi(v) = \Phi^{(0)} + \sum_{n \geq 1} \frac{1}{n!} \int v(x_1) \ldots v(x_n) \Phi^{(n)}(d(x_1, \ldots, x_n))$$
Point process modelling – Poisson clusters

\[ G_{\Phi_d}(h) = G_{\Phi_p}(G_{\Phi_e}(h|\cdot)) \]

Composition of Poisson processes:
Application - tracking groups
SSA context: eg. tracking debris clouds
Functional derivatives and the population mean

Important statistical quantities are determined from the p.g.fl. with functional derivatives:

\[ \delta f(x; \eta) = \lim_{n \to \infty} \frac{1}{\theta_n} (f(x + \theta_n \eta_n) - f(x)) \]

For example, the mean, or intensity, measure is found with

\[ \mu^{(1)}_{\Phi}(B) = \delta(\mathcal{G}_\Phi[h]; 1_B)|_{h=1}, \]
THE PHD FILTER

PHD filter [Mahler 03]

\[ \cdots \rightarrow v_{k-1|k-1}(x_{k-1}|Z_{1:k-1}) \xrightarrow{\text{PHD prediction}} v_{k|k-1}(x_k|Z_{1:k-1}) \xrightarrow{\text{PHD update}} v_{k|k}(x_k|Z_{1:k}) \rightarrow \cdots \]

\[ \cdots \rightarrow p_{k-1|k-1}(X_{k-1}|Z_{1:k-1}) \xrightarrow{\text{prediction}} p_{k|k-1}(X_k|Z_{1:k-1}) \xrightarrow{\text{update}} p_{k|k}(X_k|Z_{1:k}) \rightarrow \cdots \]

Multi-target Bayes filter
TRACKING FROM TELESCOPE DATA
To detect and track observed objects
To classify objects in the scene (e.g. stars vs satellites)
To estimate and compensate for telescope drift
TELESCOPE DRIFT

1. BACKGROUND

- Mechanical imperfections of the mount
- Diurnal motion of the stars (in case of the static mount)
- Basic jitter due to the wind or unstable earth
CURRENT SOLUTIONS

1. BACKGROUND

- Star guiding [6,7]
- Image registration [8,9]
Joint sensor estimation and multi-target tracking\cite{18}:

- **Parent process** – telescope motion
- **Daughter process** – objects motion
- Particle filter for sequential estimation of telescope position

Every particle is a hypothesis of a telescope position with linked multi-target estimation and weight.

Sensor state space
Particle filtering

Multi-object state space (PHD filter)

Observation state space
Image detections
Joint sensor estimation and multi-target tracking:

- **Parent process** – telescope motion
- **Daughter process** – objects motion

Particle filter for sequential estimation of telescope position

Every particle is a hypothesis of a telescope position with linked multi-target estimation and weight

Weight is assigned to the particles according to the likelihood of the observations, given sensor state estimate.
Every particle corresponds to:

- Sensor state estimate (relative position of the telescope)
- Multi-target state for objects (linear motion model)
- Multi-target state for stars (static)

\[
p(X_k, y_k | Z_{1:k}) = p(X_k | Z_{1:k}, y_k) p(y_k | Z_{1:k})
\]

Multi-target filter  Particle filter
REAL DATA RESULTS

(NEO 2007HA during its close passage to the Earth).
Joint estimation of telescope drift and object tracking

NEO 2007HA during its close passage
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