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Direction Finding Antenna Arrays with Improved Accuracy and Reduced Complexity and Size

Houcem Gazzah
Dept. of Elec. and Computer Engineering
University of Sharjah, 27272, UAE
hgazzah@sharjah.ac.ae

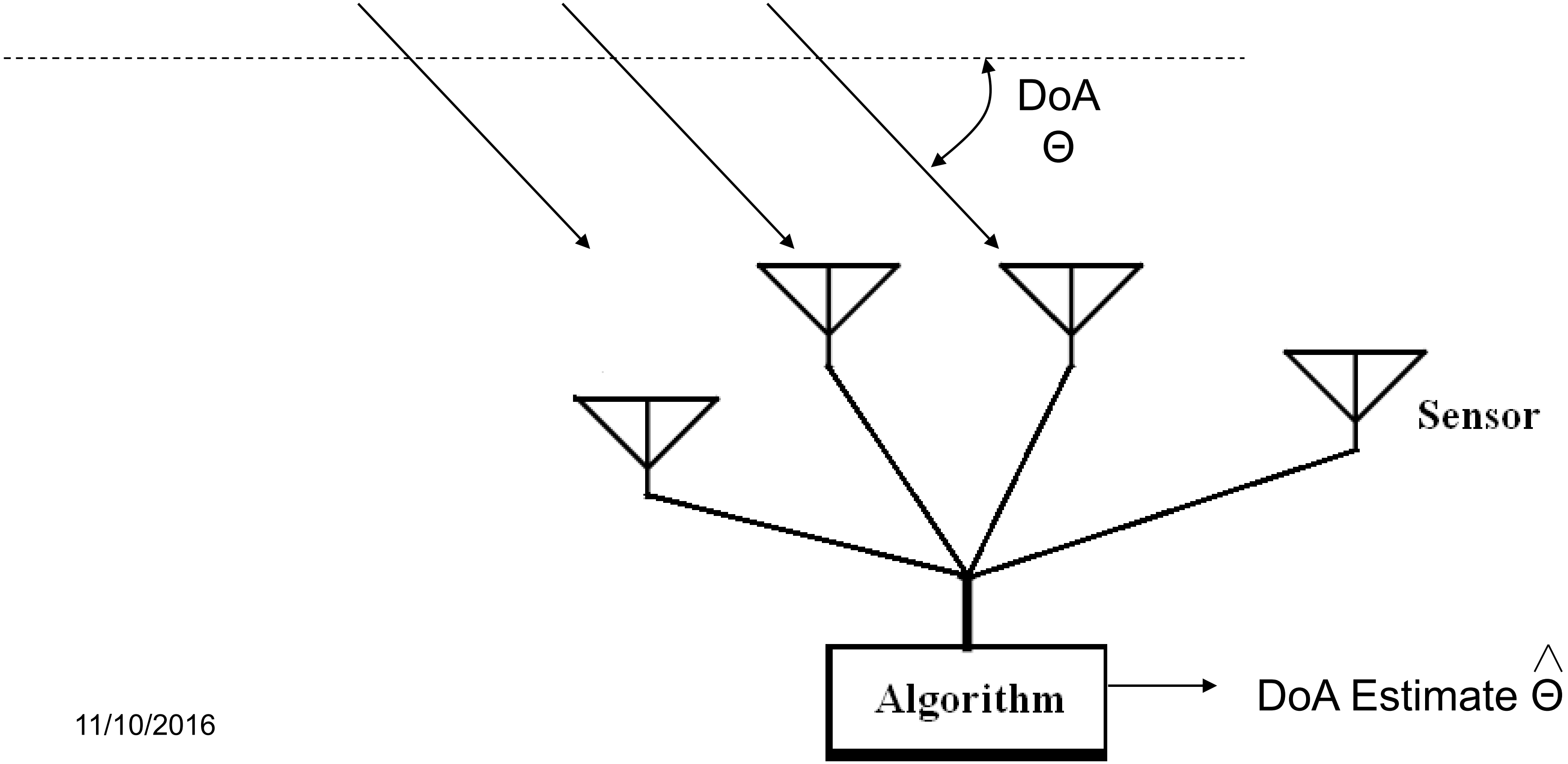
Background

•Estimating the Direction of Arrival **(DOA) of an emitting source**
•Major topic of statistical signal applications
•Important civil and military applications
•Huge literature

•Most **algorithms are equally efficient** in the single source case
•**Sensor placement has a larger impact** on accuracy
•Improvement is exacerbated **if sensors are (more) directive**
•Approach **applied to** far/near field deterministic/random sources and small/large sized arrays of omni/directional sensors

2D Direction Finding

- Far-field source emitting a narrow-band signal
- Signal collected using an array of (omni) directional sensors
- Signal phase (and amplitude) depends on source DOA



CRB-Based Design

Accuracy is evaluated in terms of the **Cramer-Rao Bound**

•It is algorithm-independent and is achievable in practice

•It is different from a look direction to an other

Expected CRB

•An overall performance measure

•Allows the CRB to be high at look directions that are less probable

•**ECRB depends on the source PDF** (and the array geometry)

Array geometry optimization

•Based on the available (statistical) information about the source

•We compare **ECRB of the flexible-geometry array to ECRB of the fixed-geometry UCA**

Previous work: Arrays of omni-directional sensors

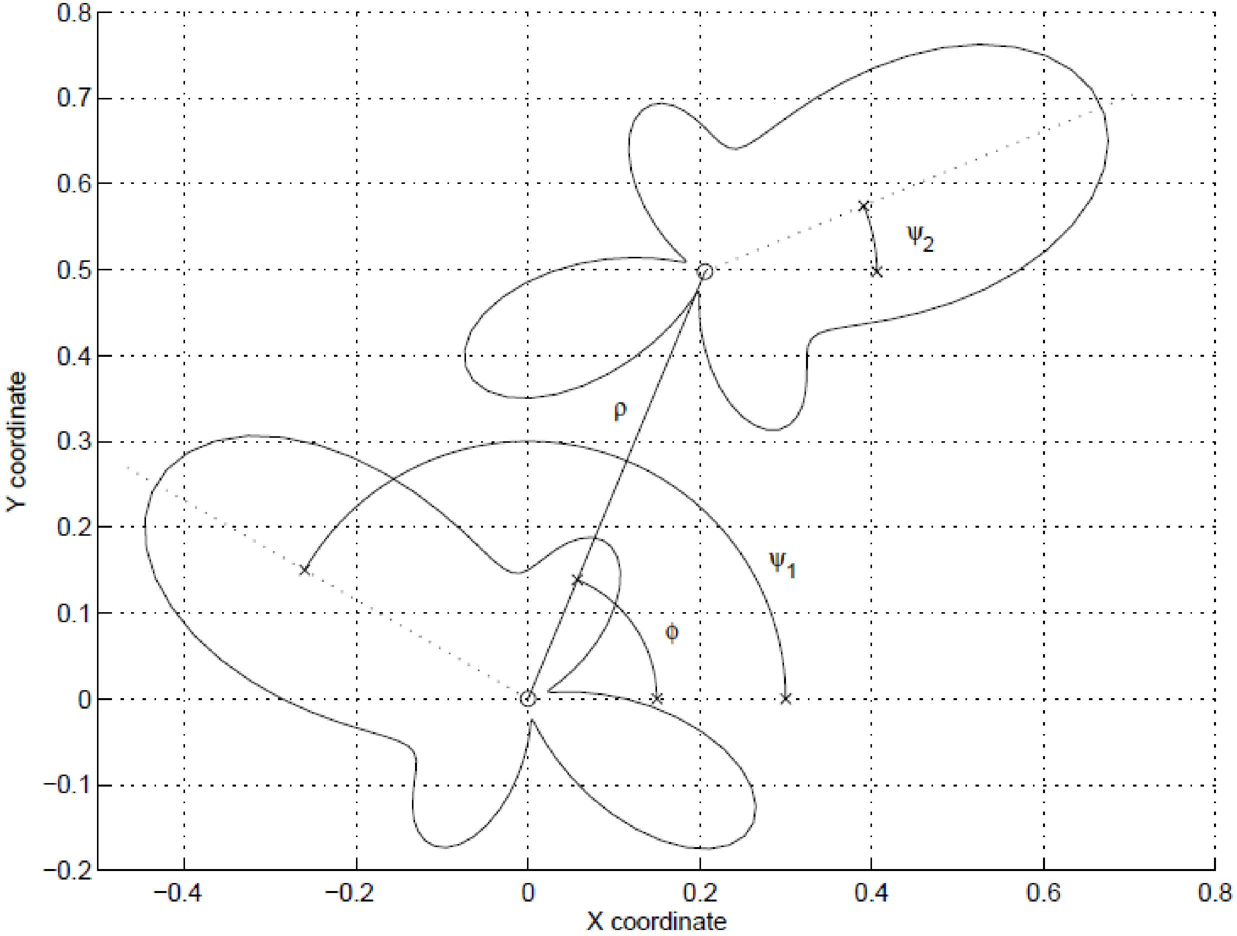
- Simple expression of CRB/ECRB allows for closed-form solutions
- 36% CRB reduction, simultaneously at all look directions
- Up to 85% ECRB reduction depending on source distribution

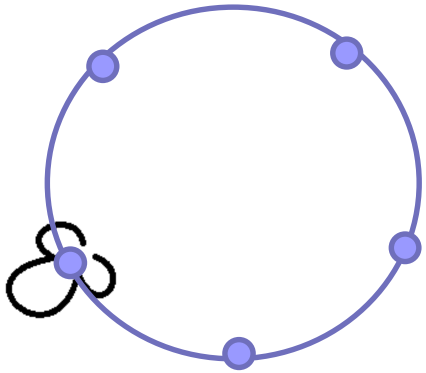
Current work: Arrays of directional sensors

- Complex expression of the ECRB implies
  - No attractive algorithm to calculate the optimal geometry
  - Systematic search is affordable for minimally-sized (AUVs)
- Two-sensors arrays
  - ECRB-minimization requires 3D search
  - Near ECRB-minimization requires 2D search
  - Globally, the two-sensor array is as accurate as UCA of 3 or 4 sensors

Geometry of the Two-Sensor Array

- Inter-sensor spacing left to jugulate ambiguity/coupling
- Three angular parameters to be determined by systematic search
- Compare flexible **two-sensor array** to **UCA of 3,4,5** sensors





Signal Model

$$\mathbf{x}(t) = \begin{bmatrix} g(\theta-\psi_1) \\ g(\theta-\psi_2)\exp\left[j2\pi\rho\cos\left(\theta-\phi_m\right)\right] \end{bmatrix} s(t) + \begin{bmatrix} n_1(t) \\ n_2(t) \end{bmatrix}$$

$$\mathrm{CRB}(\theta) = \frac{\sigma_n^2}{2N\sigma_s^2} F^{-1}(\theta)$$

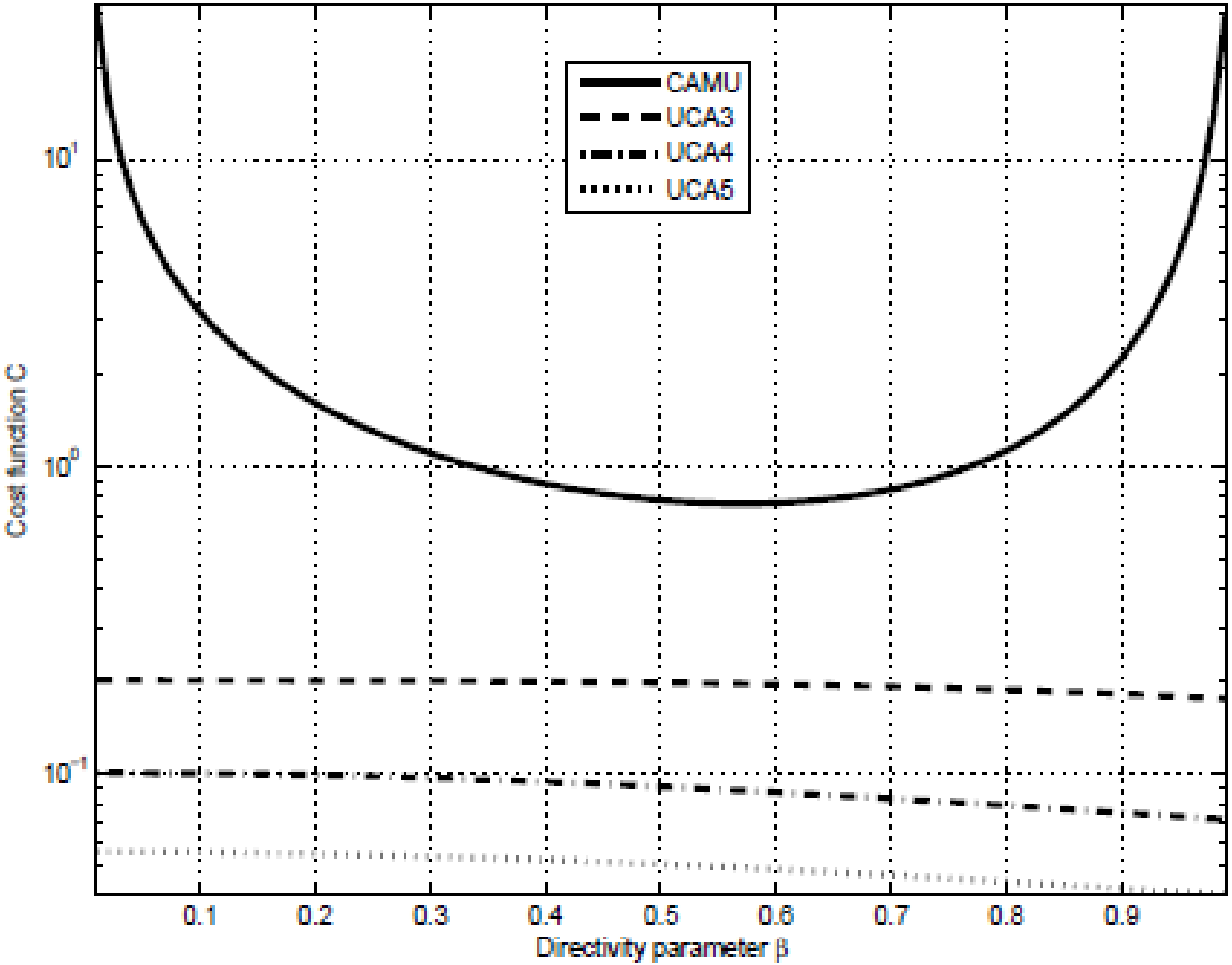
$$F(\theta) = \frac{\left[\frac{g'(\theta-\psi_1)}{g(\theta-\psi_1)} - \frac{g'(\theta-\psi_2)}{g(\theta-\psi_2)}\right]^2 + 4\pi^2\rho^2\sin^2\left(\theta-\phi\right)}{\frac{1}{g^2(\theta-\psi_1)} + \frac{1}{g^2(\theta-\psi_2)}}$$

The case of cardioid sensors $g(\theta) = g_0[1+\beta\cos(\theta)],$

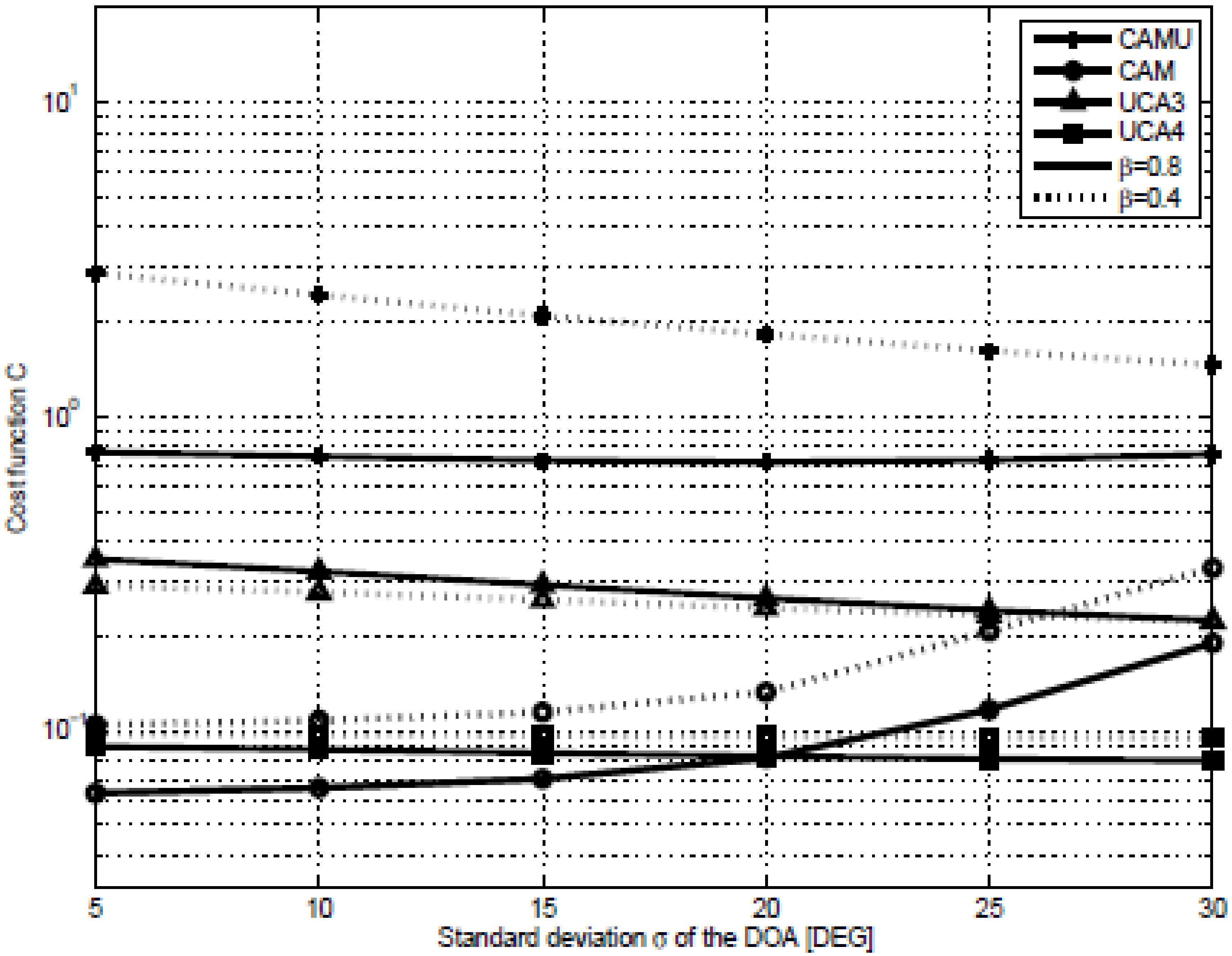
$$\frac{F(\theta)}{g_0^2} = \frac{\beta^2\left[\frac{\sin(\theta-\psi_1)}{1+\beta\cos(\theta-\psi_1)} - \frac{\sin(\theta-\psi_2)}{1+\beta\cos(\theta-\psi_2)}\right]^2 + 4\pi^2\rho^2\sin^2(\theta-\phi)}{\frac{1}{[1+\beta\cos(\theta-\psi_1)]^2} + \frac{1}{[1+\beta\cos(\theta-\psi_2)]^2}}$$

$$C \hat{=} \int_{-\pi}^{\pi} \frac{\left(\frac{1}{[1+\beta\cos(\theta-\psi_1)]^2} + \frac{1}{[1+\beta\cos(\theta-\psi_2)]^2}\right) f(\theta)}{\beta^2\left[\frac{\sin(\theta-\psi_1)}{1+\beta\cos(\theta-\psi_1)} - \frac{\sin(\theta-\psi_2)}{1+\beta\cos(\theta-\psi_2)}\right]^2 + 4\pi^2\rho^2\sin^2\left(\theta-\phi\right)} d\theta$$

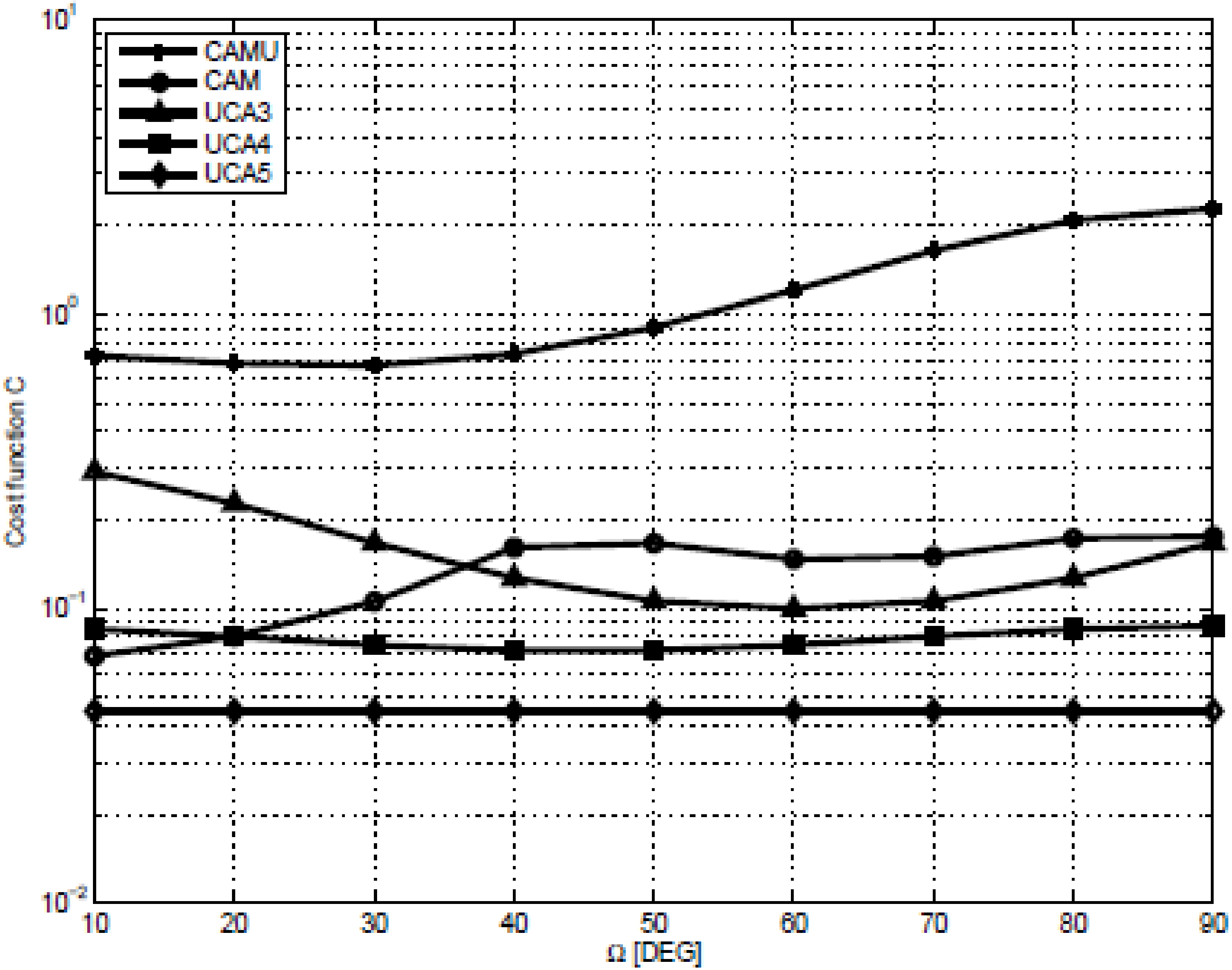
Results if No Prior



Results if Gaussian Prior



Results if Arbitrary Prior



An Alternative Criterion

Strictly speaking, (the expected) **Fisher-information** is not physically-relevant. However, it **depends on parameter Φ in a simple manner**, leading to **closed-form** expression of optimum Φ

$$\mathrm{E}\left[F(\theta)\right] = I_0 - 2\pi^2\rho^2\left[\cos(2\phi)\, I_1 + \sin(2\phi)\, I_2\right]$$

where

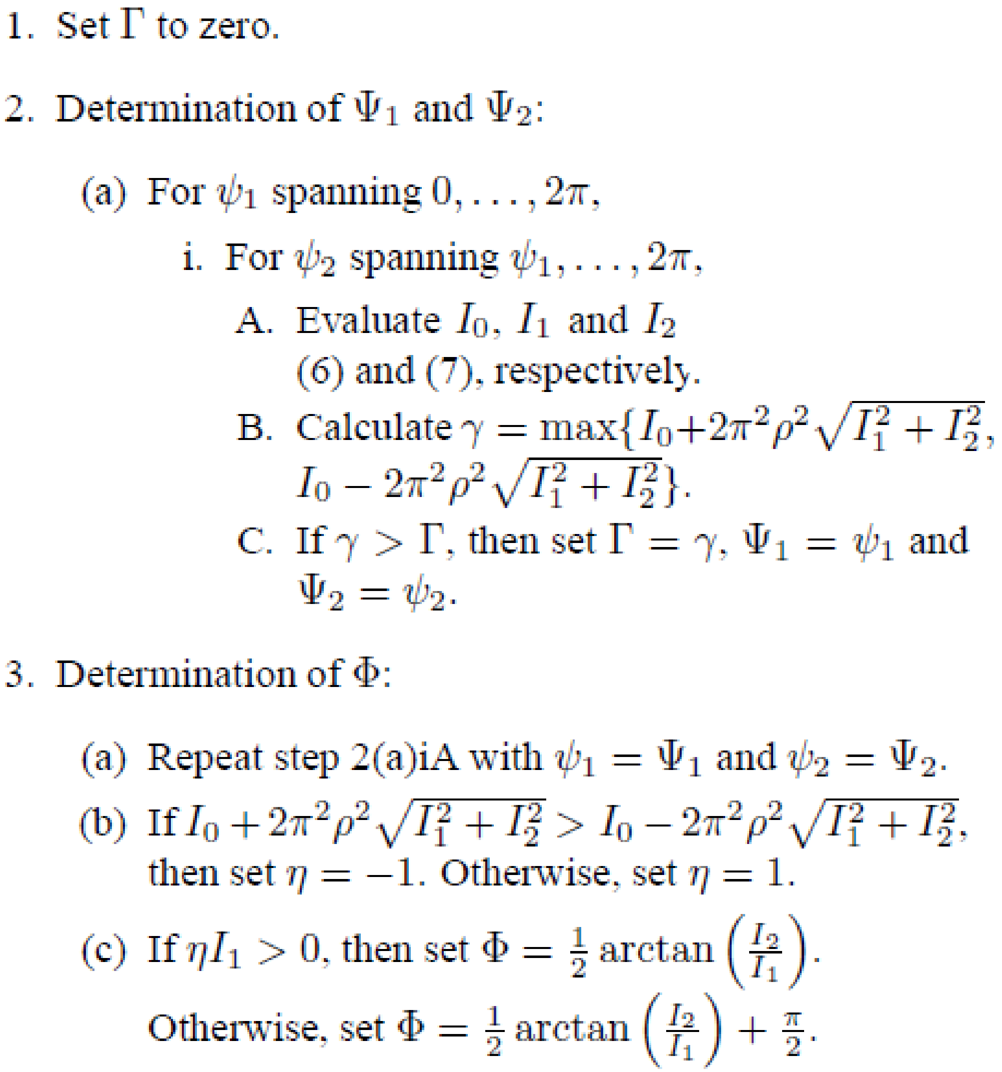
$$I_0 = \mathrm{E}\left[\frac{\left[\frac{g'(\theta-\psi_1)}{g(\theta-\psi_1)} - \frac{g'(\theta-\psi_2)}{g(\theta-\psi_2)}\right]^2 + 2\pi^2\rho^2}{\frac{1}{g^2(\theta-\psi_1)} + \frac{1}{g^2(\theta-\psi_2)}}\right]$$

$$I_1 = \mathrm{E}\left[\frac{\cos(2\theta)}{\frac{1}{g^2(\theta-\psi_1)} + \frac{1}{g^2(\theta-\psi_2)}}\right]$$

$$I_2 = \mathrm{E}\left[\frac{\sin(2\theta)}{\frac{1}{g^2(\theta-\psi_1)} + \frac{1}{g^2(\theta-\psi_2)}}\right]$$

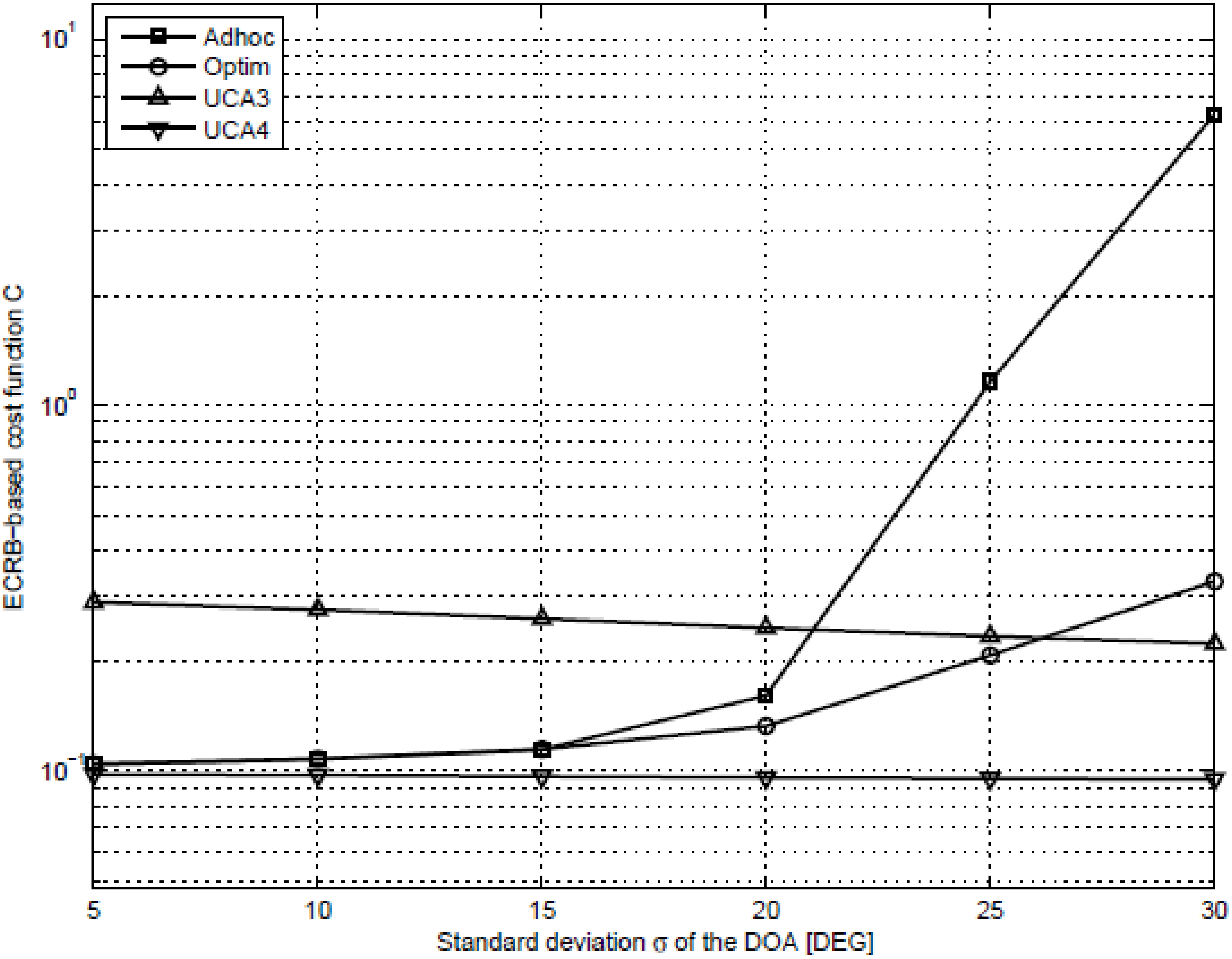
at optimality, we have $\tan(2\phi) = \frac{I_2}{I_1}$

Algorithm

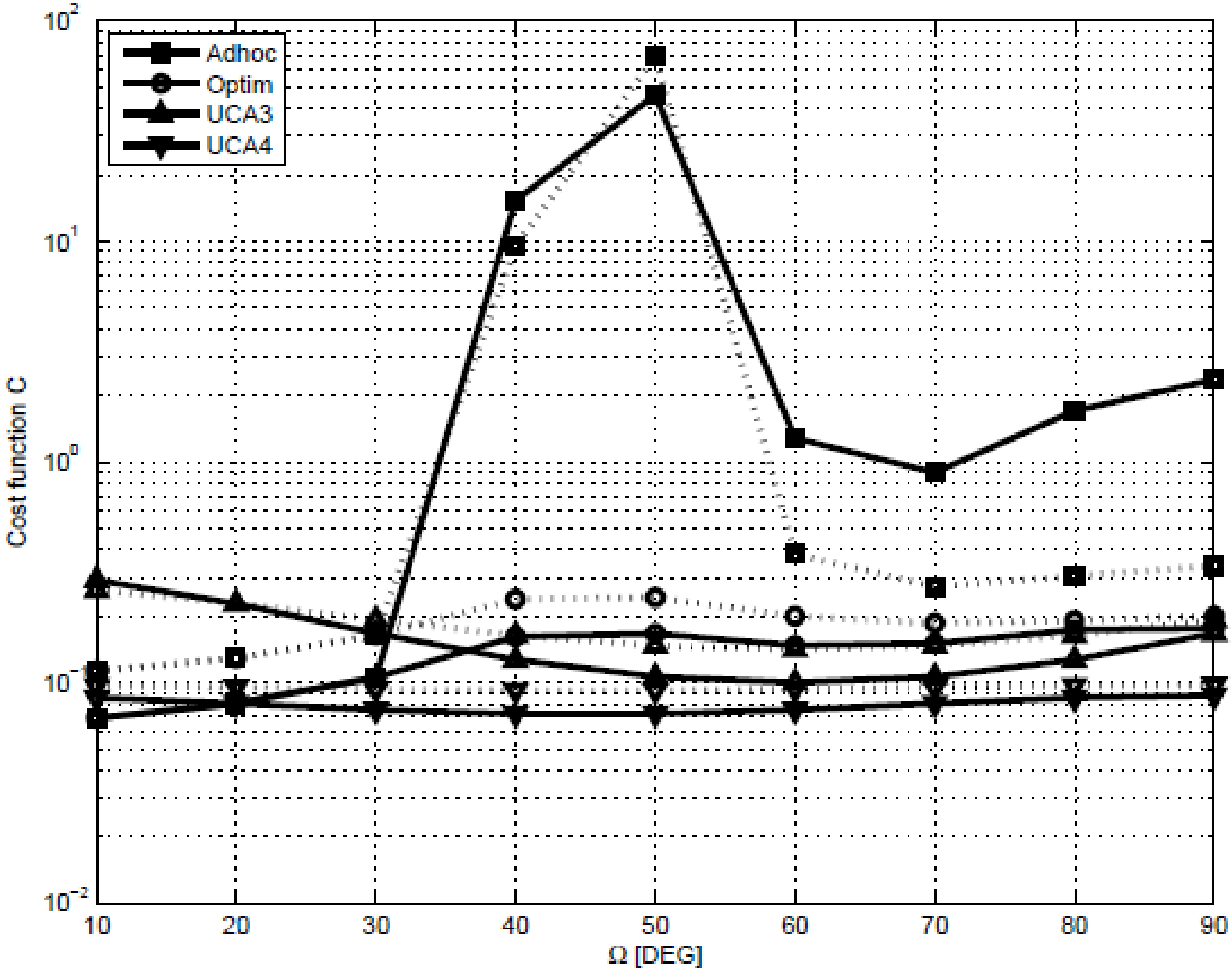


1. Set Γ to zero.
2. Determination of Ψ_1 and Ψ_2:
   - (a) For ψ_1 spanning $0, \ldots, 2\pi$,
     - i. For ψ_2 spanning $\psi_1, \ldots, 2\pi$,
       - A. Evaluate I_0, I_1 and I_2 (6) and (7), respectively.
       - B. Calculate $\gamma = \max\{I_0 + 2\pi^2\rho^2\sqrt{I_1^2 + I_2^2}, I_0 - 2\pi^2\rho^2\sqrt{I_1^2 + I_2^2}\}$.
       - C. If $\gamma > \Gamma$, then set $\Gamma = \gamma$, $\Psi_1 = \psi_1$ and $\Psi_2 = \psi_2$.
3. Determination of Φ:
   - (a) Repeat step 2(a)iA with $\psi_1 = \Psi_1$ and $\psi_2 = \Psi_2$.
   - (b) If $I_0 + 2\pi^2\rho^2\sqrt{I_1^2 + I_2^2} > I_0 - 2\pi^2\rho^2\sqrt{I_1^2 + I_2^2}$, then set $\eta = -1$. Otherwise, set $\eta = 1$.
   - (c) If $\eta I_1 > 0$, then set $\Phi = \frac{1}{2}\arctan\left(\frac{I_2}{I_1}\right)$. Otherwise, set $\Phi = \frac{1}{2}\arctan\left(\frac{I_2}{I_1}\right) + \frac{\pi}{2}$.

Performance at normal prior



Performance at arbitrary prior



Performance comparison for a source PDF characterized by two possible look directions $\pm\Omega$, with $\Omega = 10, 20, \ldots, 90$ [DEG]. Sensors are such that $\beta = 0.4$ (dotted line) and $\beta = 0.8$ (solid line).

Conclusion

- Array geometry optimization allows for **better source localization**

- Improvement increases with the sensor **directivity**

- Suitable **for minimally-sized arrays** because of the computation load

- An **adhoc** criterion **reduces the number of unknowns** from 3 to 2 and maintains **near optimum performance** unless prior information is weak