A Multi-Family GLRT for Detection in Polarimetric SAR Images

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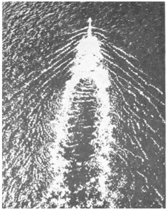
Introduction

- **Polarimetric SAR images** provide enhanced information on the imaged scene that can be exploited for improved target detection, recognition and scene classification;
- **Target detection** can be improved exploiting the multi-polarimetric nature of the data;
- We formulate the problem of target detection in terms of a **binary hypothesis test** aimed at discriminating between the presence and the absence of variations in the **Polarimetric Covariance Matrix (PCM)** of the radar returns;

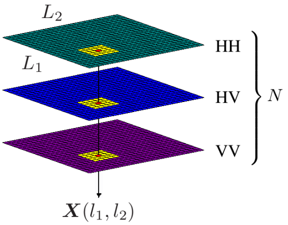
Introduction- Contd.



- We are interested in targets such as ship wakes or oil spills that modify the backscattering of sea surface;
- We will focus on the specific case of oil spills, where it is reasonable to assume that the PCM of data containing oil slicks **share eigenvalues** smaller than or equal to the PCM of the sea returns;
- The decision problem is solved applying the GLRT obtaining an architecture called **Positive Definite Difference GLRT (PDD-GLRT).**

Problem Formulation

- The N returns associated with the same pixel are organized in the specific order HH, HV, and VV to form the vector $\boldsymbol{X}(l_1, l_2)$, $l_1 = 1, \ldots, L_1$ and $l_2 = 1, \ldots, L_2$. The sensor provides a **3-D data stack** $\boldsymbol{X}$ of size $L_1 \times L_2 \times N$.



- Starting from the datacube $\boldsymbol{X}$ of the scene illuminated by the radar, for each pixel under test, we extract a rectangular neighborhood $\mathcal{A}$ of size $K = W_1 \times W_2 \geq N$. $\boldsymbol{Z} = [\boldsymbol{z}_1 \ \ldots \ \boldsymbol{z}_K] \in \mathbb{C}^{N \times K}$ is the matrix whose columns are the vectors of the polarimetric returns from the pixels of $\boldsymbol{X}$ which fall in the region $\mathcal{A}$.
- The matrix $\boldsymbol{Z}$ is modeled as a random matrix, whose columns are assumed **iid random vectors drawn from a complex circular zero-mean Gaussian distribution** with positive definite covariance matrix $\boldsymbol{R}$.

Problem Formulation

The goal is to identify those regions of $\boldsymbol{X}$ that **exhibit variations of the covariance matrix** $\boldsymbol{R}$ with respect to a preassigned reference region.

The detection problem can be written in terms of the following **hypothesis test**

$$\begin{cases} H_0 : \begin{cases} \boldsymbol{z}_k \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{R}) & k = 1, \ldots, K \\ \boldsymbol{y}_m \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{R}) & m = 1, \ldots, M \end{cases} \\ H_1 : \begin{cases} \boldsymbol{z}_k \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{R}_1) & k = 1, \ldots, K \\ \boldsymbol{y}_m \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{R}_1 + \boldsymbol{R}_2) & m = 1, \ldots, M \end{cases} \end{cases}$$

where

- $\boldsymbol{z}_k$, $k = 1, \ldots, K$, and $\boldsymbol{y}_m$, $m = 1, \ldots, M$, are statistically independent random vectors;
- matrices $\boldsymbol{R}$ and $\boldsymbol{R}_1$ are **full-rank,** namely $\mathrm{Rank}\,(\boldsymbol{R}) = \mathrm{Rank}\,(\boldsymbol{R}_1) = N$;
- the rank of $\boldsymbol{R}_2$, say p, is assumed known and within the interval $(0,\ N]$.

Moreover, we assume that $K \geq N$ and $M \geq N$ to ensure that the PCMs

$$\boldsymbol{G} = \sum_{k=1}^{K} \boldsymbol{z}_k \boldsymbol{z}_k^\dagger \quad \text{and} \quad \boldsymbol{H} = \sum_{m=1}^{M} \boldsymbol{y}_m \boldsymbol{y}_m^\dagger$$

are full-rank with probability 1.

Detector Design

An adaptive decision rule is devised resorting to the **GLRT design criterion.** To this end, let us define $\boldsymbol{Z}_K = [\boldsymbol{z}_1 \ \ldots \ \boldsymbol{z}_K]$ and $\boldsymbol{Y}_M = [\boldsymbol{y}_1 \ \ldots \ \boldsymbol{y}_M]$, then the likelihood functions of $\boldsymbol{Z}_K$ and $\boldsymbol{Y}_M$ under H_0 and H_1 are given by

$$f(\boldsymbol{Z}_K, \boldsymbol{Y}_M; \boldsymbol{R}, H_0) = \left[\frac{1}{\pi^N \det(\boldsymbol{R})}\right]^{K+M} \exp\left\{-\mathrm{tr}\left[\boldsymbol{R}^{-1}(\boldsymbol{G}+\boldsymbol{H})\right]\right\}$$

and

$$f(\boldsymbol{Z}_K, \boldsymbol{Y}_M; \boldsymbol{R}_1, \boldsymbol{R}_2, H_1) = \left[\frac{1}{\pi^N}\right]^{K+M} \frac{\exp\left\{-\mathrm{tr}\left[\boldsymbol{R}_1^{-1}\boldsymbol{G}\right] - \mathrm{tr}\left[(\boldsymbol{R}_1+\boldsymbol{R}_2)^{-1}\boldsymbol{H}\right]\right\}}{[\det(\boldsymbol{R}_1)]^K [\det(\boldsymbol{R}_1+\boldsymbol{R}_2)]^M}.$$

The GLRT for the problem at hand is

$$\frac{\max\limits_{\boldsymbol{R}_1} \max\limits_{\boldsymbol{R}_2} f(\boldsymbol{Z}_K, \boldsymbol{Y}_M; \boldsymbol{R}_1, \boldsymbol{R}_2, H_1)}{\max\limits_{\boldsymbol{R}} f(\boldsymbol{Z}_K, \boldsymbol{Y}_M; \boldsymbol{R}, H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

Detector Design

It is possible to show that the GLRT is statistically equivalent to

$$\Lambda_p(\boldsymbol{Z}_K, \boldsymbol{Y}_M) \underset{H_0}{\overset{H_1}{\gtrless}} \eta,$$

where

$$\Lambda_p(\boldsymbol{Z}_K, \boldsymbol{Y}_M) = \begin{cases} 1, & \text{if } p^* < p, \\ \displaystyle\prod_{i=1}^{p^*} \frac{(1+\delta_i)^{K+M}}{\delta_i^M}, & \text{otherwise,} \end{cases}$$

δ_i, $i = 1, \ldots, p^*$, are p^* eigenvalues of $\boldsymbol{G}^{-1}\boldsymbol{H}$ with p^* being the minimum between the number of eigenvalues of $\boldsymbol{G}^{-1}\boldsymbol{H}$ greater than M/K and p.

This detector is the **PDD-GLRT.**

Multi-Family PDD-GLRT

- As the rank of $\boldsymbol{R}_2$ is generally unknown, the test has **multiple nested instances**;
- In this scenario, the classical GLRT cannot be used and, hence, we resort to the **Multifamily GLRT (MGLRT)**

The MGLRT can be written in terms of the Exponentially Embedded Families (EEF) computed for a given model order i, $i = 1, \ldots, N$, namely

$$\max_{i \in \{1,\ldots,N\}} \text{EEF}(i) \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

where

$$\text{EEF}(i) = \left(\zeta_{G_i}(\boldsymbol{Z}_K, \boldsymbol{Y}_M) - r(i) \left[\log \left(\frac{\zeta_{G_i}(\boldsymbol{Z}_K, \boldsymbol{Y}_M)}{r(i)} \right) + 1 \right] \right)$$
$$\mathrm{u} \left(\frac{\zeta_{G_i}(\boldsymbol{Z}_K, \boldsymbol{Y}_M)}{r(i)} - 1 \right), \quad i = 1, \ldots, N.$$

$r(i) = i$, $i = 1, \ldots, N$ is the rank of $\boldsymbol{R}_2$, $\mathrm{u}(\cdot)$ is the Heaviside step function and $\zeta_{G_i}(\boldsymbol{Z}_K, \boldsymbol{Y}_M) = 2 \log \Lambda_i(\boldsymbol{Z}_K, \boldsymbol{Y}_M)$.

Performance Analyses

We investigate the performances of the proposed detectors in terms of **Probability of Detection (P_D)**. For comparison purposes, we also consider the GLRT, the Maximum Likelihood Detector (MLD), and the Single Likelihood Detector (SLD)

$$\Lambda_{\text{GLRT}} = \frac{[\det(\boldsymbol{G}+\boldsymbol{H})]^{(K+M)}}{[\det(\boldsymbol{G})]^{K}[\det(\boldsymbol{H})]^{M}}, \quad \Lambda_{\text{MLD}} = \frac{\det(\boldsymbol{H})}{\det(\boldsymbol{G})}, \quad \Lambda_{\text{SLD}} = \operatorname{tr}\left(\boldsymbol{G}^{-1}\boldsymbol{H}\right).$$

The comparisons also include the **clairvoyant counterparts**, namely the Likelihood Ratio Test (LRT) and the clairvoyant SLD (C-SLD):

$$\Lambda_{\text{LRT}} = \operatorname{tr}[\boldsymbol{R}^{-1}(\boldsymbol{G}+\boldsymbol{H}) - \boldsymbol{R}_1^{-1}\boldsymbol{G} - (\boldsymbol{R}_1+\boldsymbol{R}_2)^{-1}\boldsymbol{H}], \quad \text{and} \quad \Lambda_{\text{C-SLD}} = \operatorname{tr}\left(\boldsymbol{R}_1^{-1}\boldsymbol{H}\right).$$

Simulated Data

Fully polarimetric SAR is considered and the nominal P_{FA} is set to 10^{-4}.

Matrix $\boldsymbol{R}_2$ is rank deficient and is defined as $\boldsymbol{R}_2 = |\alpha|^2 \boldsymbol{p}_1 \boldsymbol{p}_1^\dagger + |\beta|^2 \boldsymbol{p}_2 \boldsymbol{p}_2^\dagger$, where the N-dimensional steering vectors, $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$, have been chosen as

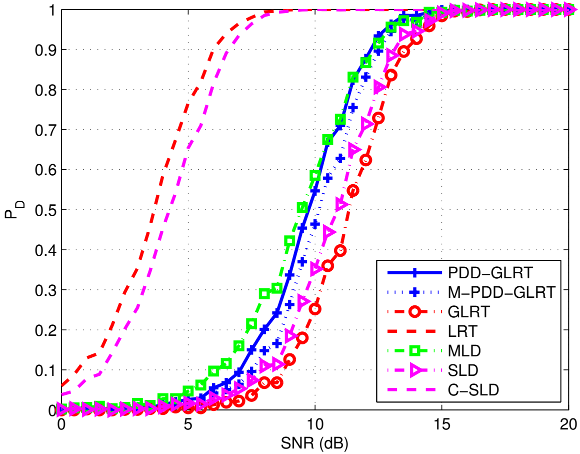
$$\boldsymbol{p}_1 = [1, 0, \ldots, 0]^T, \quad \boldsymbol{p}_2 = [0, 1, 0, \ldots, 0]^T,$$

and $|\alpha|^2 = |\beta|^2$, so as $\boldsymbol{R}_2$ is a rank-2 matrix. The **SNR** is defined as

$$\text{SNR} = |\alpha|^2 \boldsymbol{p}_1^\dagger \boldsymbol{R}_1^{-1} \boldsymbol{p}_1 + |\beta|^2 \boldsymbol{p}_2^\dagger \boldsymbol{R}_1^{-1} \boldsymbol{p}_2 = 2|\alpha|^2.$$

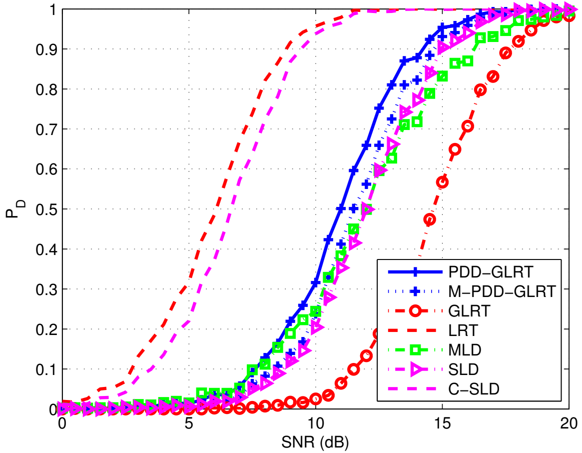
Simulated Data

P_D **versus SNR.** The simulation parameters are $K = 9$, $M = 9$, $N = 3$, and $p = 2$.



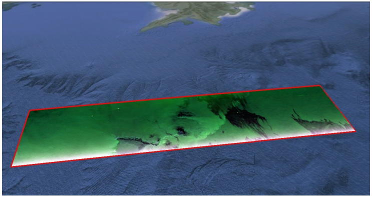
Simulated Data

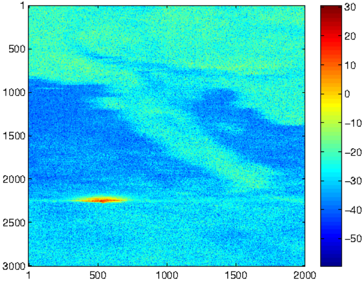
P_D **versus SNR.** The simulation parameters are $K = 9$, $M = 4$, $N = 3$, and $p = 2$.



Test on Real SAR Data

We use a data set from the UAVSAR L-BAND Polarimetric SAR sensor acquired during the BP oil spill in 2010 in the Gulf of Mexico.

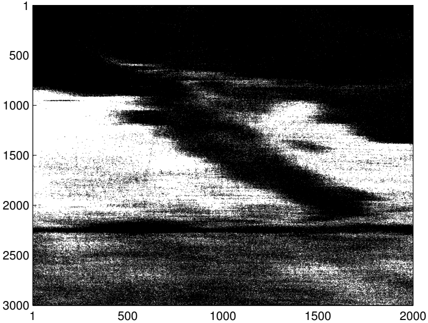




Test on Real Radar Data

A sea data pixel is chosen as reference to compute the sample covariance $\boldsymbol{H}$ from a 3×3 (i.e., $M = 9$) window centered in that pixel. Then, a window of size 3×3 (i.e., $K = 9$) is slided over the SAR image to test all the pixels and to compute the sample matrix $\boldsymbol{G}$.

The **threshold** is chosen to ensure a nominal P_{FA} of 10^{-3}.



Conclusions

- Multi-polarization SAR detection has been considered to **test the equality of two polarimetric sample covariance matrices** constructed from a reference area and a test region;
- The alternative hypothesis is represented by the instance where the PCM within the reference area exhibits at least **an ordered eigenvalue greater** than the corresponding one extracted from the PCM of the area under test;
- A **PDD-GLRT** has been obtained assuming the exact knowledge about the number of different eigenvalues between the reference and tested PCMs;
- A fully adaptive detector without any a priori assumption about the number of different eigenvalues has been then obtained, the **M-PDD-GLRT**;
- The proposed detector showed higher robustness and practical applicability than other existing detectors.

Possible **future research tracks** might concern:

- The evaluation of performance in other cases of interest;
- The extension of the approach to the case of a joint multi-frequency and multi-polarization processing;
- Presence of a non-Gaussian backscattering.

THANKS FOR YOUR ATTENTION