Radar Filters Design in the Presence of Target Doppler Frequency and Interference Covariance Matrix Uncertainties

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ELETTRONICA mind is the first defence

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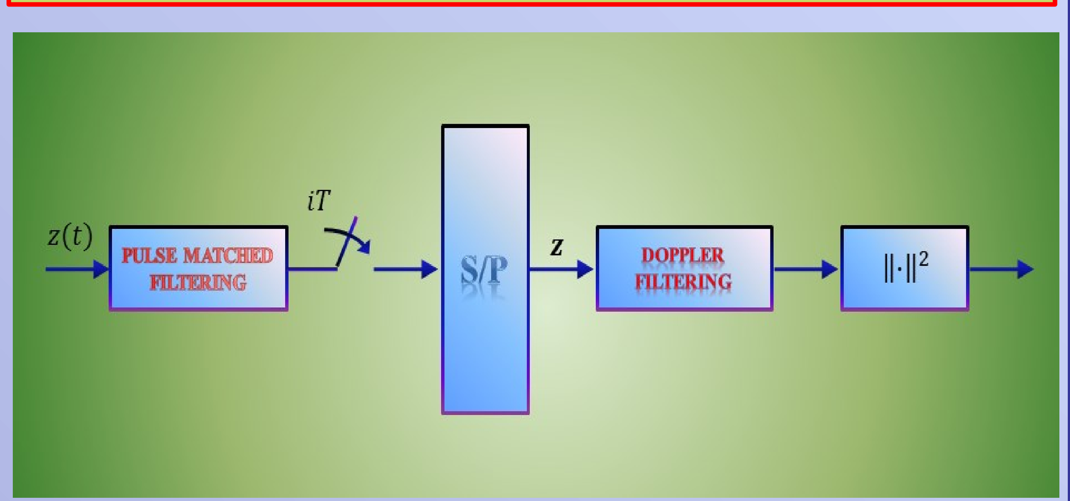
ABSTRACT



The design of robust filters for radar pulse-Doppler processing is considered.

- **GOAL**: optimize the **worst-case Signal-to-Interference-plus-Noise Ratio (SINR)**.
- **UNCERTAINTIES**:
  - **useful target signature**,
  - **interference second-order statistics**.
- **CONSTRAINTS**: **Doppler filter sidelobe** levels.
- **TECHNICAL CONTRIBUTION**: development of a polynomial-time solution technique to get the optimal robust filter exploiting:
  - **spectral factorization theorem,**
  - **non-negative trigonometric polynomials features,**
  - **duality theory.**
- **PERFORMANCE ASSESSMENT**: detailed analysis of the optimal robust filter behavior in terms of:
  - **trade-offs** involved in the design,
  - **gain achievable** over known counterparts.

SECTION I: INTRODUCTION & SIGNAL MODEL

KEY ASPECTS OF DOPPLER PROCESSING

The design of **optimized Doppler processors in coherent pulsed radar** has been a **hot research activity** within the radar community.

⇓

- **mitigate the interference** competing with the useful signal ⇒ **increase the detection capabilities** of targets embedded in heavy clutter
- **provide measurements** of the target radial velocity ⇒ **threat assessment** and possible **track initialization**

SIGNAL MODEL

$$\boldsymbol{z} = \alpha \boldsymbol{s}(\nu) + \boldsymbol{n}$$

- $\boldsymbol{z} \in \mathbb{C}^N$: **slow-time** observations from the cell under test;
- $\boldsymbol{s}(\nu) \in \mathbb{C}^N$: **target steering vector**;
- $\nu \in \mathbb{R}$: normalized target **Doppler frequency**;
- $\alpha \in \mathbb{C}$: related to the **target backscattering**;
- $\boldsymbol{n} \in \mathbb{C}^N$: zero-mean, **circularly symmetric Wide-Sense Stationary** (**WSS**) **interference** with covariance matrix $\boldsymbol{R}_0$.

In a Doppler processor, the data vector $\boldsymbol{z}$ **undergoes a specific linear transformation defined by the simple Doppler filter** $\boldsymbol{w} \in \mathbb{C}^N$.
A widely used figure of merit to **assess the performance of a Doppler filter** $\boldsymbol{w}$ is the **SINR**

$$\mathrm{SINR}(\nu, \boldsymbol{R}_0) = |\alpha|^2 \frac{|\boldsymbol{w}^\dagger \boldsymbol{s}(\nu)|^2}{\boldsymbol{w}^\dagger \boldsymbol{R}_0 \boldsymbol{w}}$$

Useful Energy

Interference Energy

SOME POSSIBLE FILTER DESIGN STRATEGIES

FILTER	HYPOTHESIS & DEF.	PROS/CONS
Capon	Doppler: known (ν) Interf. Covariance: known ($\boldsymbol{R}_0$) Filter: $\boldsymbol{w} = \boldsymbol{R}_0^{-1}\boldsymbol{s}(\nu)$	Pros: max. SINR Cons: ν and $\boldsymbol{R}_0$ unknown in practice
SMI	Doppler: nominal value ($\bar{\nu}$) Interf. Covariance: sample covariance matrix ($\hat{\boldsymbol{R}}$) Filter: $\boldsymbol{w} = \hat{\boldsymbol{R}}^{-1}\boldsymbol{s}(\bar{\nu})$	Pros: adaptive version of the Capon, easy Cons: sensible loss in case of mismatches
Other advanced filtering techniques	Doppler/Interf. Covariance: specific uncertainty sets are considered Filter: depends on the technique (specific algorithms are devised)	Pros: robustness with respect to possible non-idealities Cons: complexity, the uncertainty models might not be accurate in some specific scenarios

SECTION II: PROBLEM FORMULATION

The following max-min optimization problem, addressing the **worst-SINR maximization**, is considered

$$\mathcal{P}_1 \begin{cases} \max\limits_{\boldsymbol{w}} \min\limits_{\boldsymbol{R},\boldsymbol{s}} & \dfrac{|\boldsymbol{w}^\dagger \boldsymbol{s}|^2}{\boldsymbol{w}^\dagger \boldsymbol{R} \boldsymbol{w}} \\ \quad\text{s.t.} & \boldsymbol{R} \in \Omega \\ & \boldsymbol{s} \in \mathcal{S} \\ & \boldsymbol{w} \in \mathcal{H} \end{cases}$$

where

- Ω is a **convex compact subset of positive definite Toeplitz matrices**, accounting for the **uncertainty over the disturbance covariance matrix**;
- $\mathcal{S} = \left\{\boldsymbol{s} \in \mathbb{C}^N : \boldsymbol{s} = \left[1, e^{j2\pi\nu}, \ldots, e^{j2\pi(N-1)}\right]^T, \nu \in [\nu_1, \nu_2]\right\}$ is the set of **potential steering vectors** $\boldsymbol{s}$, accounting for the **uncertainty on the parameter ν which lies within $\Theta_1 = [\nu_1, \nu_2]$**;
- $\mathcal{H} = \left\{\boldsymbol{w} \in \mathbb{C}^N : \max_{\nu \in \Theta_2} |\boldsymbol{w}^\dagger \boldsymbol{s}(\nu)|^2 \le \zeta \min_{\nu \in \Theta_1} |\boldsymbol{w}^\dagger \boldsymbol{s}(\nu)|^2\right\}$ is the **set of receive filters** $\boldsymbol{w}$ whose output level in the **sidelobe regions $\Theta_2 = [\nu_3, \nu_4]$, is lower than or equal to ζ times the minimum level in Θ_1** (with $\Theta_1 \cap \Theta_2 = \emptyset$).

Robust Receive Filter Design Algorithm

Input: $\Theta_1, \Theta_2, \zeta, \Omega, N$.

Output: An optimal solution $w^\star$ to $\mathcal{P}_1$.

1. **Solve the convex Problem $\mathcal{P}_4$ ($\mathcal{P}_5$) finding an optimal solution $c^\star$.**
2. **Exploit $c^\star$ to find a $\bar{w}$ optimal to Problem $\mathcal{P}_2$.**
3. **Output $w^\star = \bar{w}$ optimal to Problem $\mathcal{P}_1$.**

SECTION III: ROBUST DOPPLER FILTER DESIGN

$\mathcal{P}_1$

Equivalent formulation

STEP 3

$$\mathcal{P}_2 \begin{cases} \max\limits_{\boldsymbol{w}} \min\limits_{\boldsymbol{R}} & \dfrac{1}{\boldsymbol{w}^\dagger \boldsymbol{R} \boldsymbol{w}} \\ \quad\text{s.t.} & \boldsymbol{R} \in \Omega\,, \\ & |\boldsymbol{w}^\dagger \boldsymbol{s}|^2 \ge 1, \quad \forall \boldsymbol{s} \in \mathcal{S} \\ & \max\limits_{\nu \in \Theta_2} |\boldsymbol{w}^\dagger \boldsymbol{s}(\nu)|^2 \le \zeta\,. \end{cases}$$

STEP 2

STEP 1

$$\mathcal{P}_3 \begin{cases} \min\limits_{\boldsymbol{c}} \max\limits_{\boldsymbol{R}} & 2\Re\left\{\boldsymbol{c}^\dagger \boldsymbol{r}\right\} - c_0 r_0 \\ \quad s.t. & c_0 + 2\Re\left\{\sum_{k=1}^{N-1} c_k e^{-j2\pi\nu k}\right\} \ge 0, \\ & \quad \forall \nu \in [0,1] \\ & c_0 + 2\Re\left\{\sum_{k=1}^{N-1} c_k e^{-j2\pi\nu k}\right\} \ge 1, \\ & \quad \forall \nu \in [\nu_1, \nu_2] \\ & c_0 + 2\Re\left\{\sum_{k=1}^{N-1} c_k e^{-j2\pi\nu k}\right\} \le \zeta, \\ & \quad \forall \nu \in [\nu_3, \nu_4] \\ & \boldsymbol{R} \in \Omega \end{cases}$$

Application of the spectral factorization theorem

$$\mathcal{P}_4 \begin{cases} \min\limits_{\boldsymbol{c},\boldsymbol{X},\{\boldsymbol{X}_i\}_{i=1}^4} \max\limits_{\boldsymbol{R}} & 2\Re\left\{\boldsymbol{c}^\dagger \boldsymbol{r}\right\} - c_0 r_0 \\ s.t. & \boldsymbol{c} = \hat{\boldsymbol{F}}^\dagger \mathbf{diag}\left(\hat{\boldsymbol{F}} \boldsymbol{X} \hat{\boldsymbol{F}}^\dagger\right), \\ & \boldsymbol{c} - \boldsymbol{e}_1 = \hat{\boldsymbol{F}}^\dagger \mathbf{diag}\left(\hat{\boldsymbol{F}} \boldsymbol{X}_1 \hat{\boldsymbol{F}}^\dagger\right) + \\ & \qquad + \hat{\boldsymbol{F}}^\dagger\left(\boldsymbol{d} \odot \mathbf{diag}\left(\hat{\boldsymbol{F}}_1 \boldsymbol{X}_2 \hat{\boldsymbol{F}}_1^\dagger\right)\right), \\ & \zeta\boldsymbol{e}_1 - \boldsymbol{c} = \hat{\boldsymbol{F}}^\dagger \mathbf{diag}\left(\hat{\boldsymbol{F}} \boldsymbol{X}_3 \hat{\boldsymbol{F}}^\dagger\right) + \\ & \qquad + \hat{\boldsymbol{F}}^\dagger\left(\hat{\boldsymbol{d}} \odot \mathbf{diag}\left(\hat{\boldsymbol{F}}_1 \boldsymbol{X}_4 \hat{\boldsymbol{F}}_1^\dagger\right)\right), \\ & \boldsymbol{R} \in \Omega, \\ & \boldsymbol{X} \succeq \boldsymbol{0}, \boldsymbol{X}_i \succeq \boldsymbol{0}, i = 1, \ldots, 4 \end{cases}$$

Representation of non-negative trigonometric polynomials via LMIs

$$\mathcal{P}_5 \begin{cases} \min\limits_{\boldsymbol{c},\boldsymbol{X},\{\boldsymbol{X}_i\}_{i=1}^4,\{\boldsymbol{Y}_k\}_{k=0}^K} & \sum\limits_{k=1}^{K} \mathrm{tr}\left(\boldsymbol{B}_k \boldsymbol{Y}_k\right) \\ \text{s.t.} & \boldsymbol{c} = \hat{\boldsymbol{F}}^\dagger \mathbf{diag}\left(\hat{\boldsymbol{F}} \boldsymbol{X} \hat{\boldsymbol{F}}^\dagger\right) \\ & \boldsymbol{c} - \boldsymbol{e}_1 = \hat{\boldsymbol{F}}^\dagger\left(\mathbf{diag}\left(\hat{\boldsymbol{F}} \boldsymbol{X}_1 \hat{\boldsymbol{F}}^\dagger\right) + \boldsymbol{d} \odot \mathbf{diag}\left(\hat{\boldsymbol{F}}_1 \boldsymbol{X}_2 \hat{\boldsymbol{F}}_1^\dagger\right)\right) \\ & \zeta\boldsymbol{e}_1 - \boldsymbol{c} = \hat{\boldsymbol{F}}^\dagger\left(\mathbf{diag}\left(\hat{\boldsymbol{F}} \boldsymbol{X}_3 \hat{\boldsymbol{F}}^\dagger\right) + \hat{\boldsymbol{d}} \odot \mathbf{diag}\left(\hat{\boldsymbol{F}}_1 \boldsymbol{X}_4 \hat{\boldsymbol{F}}_1^\dagger\right)\right) \\ & 2\Re\{c_i\} = -\sum\limits_{k=0}^{K} \mathrm{tr}\left(\left(\boldsymbol{A}_i^k + \boldsymbol{A}_i^{k\dagger}\right)\boldsymbol{Y}_k\right), \; i = 1, \ldots, N-1 \\ & 2\Im\{c_i\} = -\sum\limits_{k=0}^{K} \mathrm{tr}\left(\left(j\boldsymbol{A}_i^k - j\boldsymbol{A}_i^{k\dagger}\right)\boldsymbol{Y}_k\right), \; i = 1, \ldots, N-1 \\ & c_0 = -\sum\limits_{k=0}^{K} \mathrm{tr}\left(\boldsymbol{A}_0^k \boldsymbol{Y}_k\right) \\ & \boldsymbol{X} \succeq \boldsymbol{0}, \boldsymbol{X}_i \succeq \boldsymbol{0}, i = 1, \ldots, 4 \\ & \boldsymbol{Y}_k \succeq \boldsymbol{0}, \; k = 0, \ldots, K \end{cases}$$

Application of duality theory

SECTION IV: PERFORMANCE ASSESSMENT

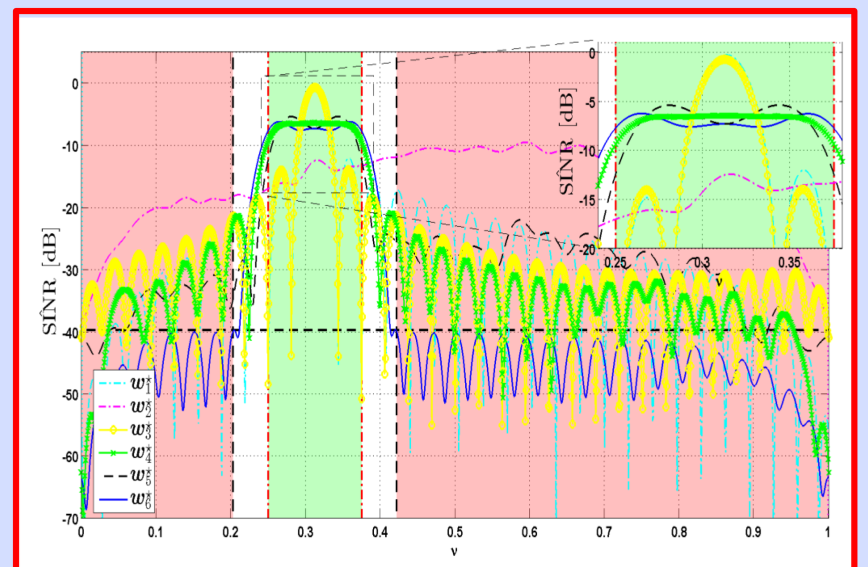
The comparison of the analyzed Doppler filters is conducted in terms of the average SINR values over the normalized Doppler frequencies of interest.

Simulation Parameters

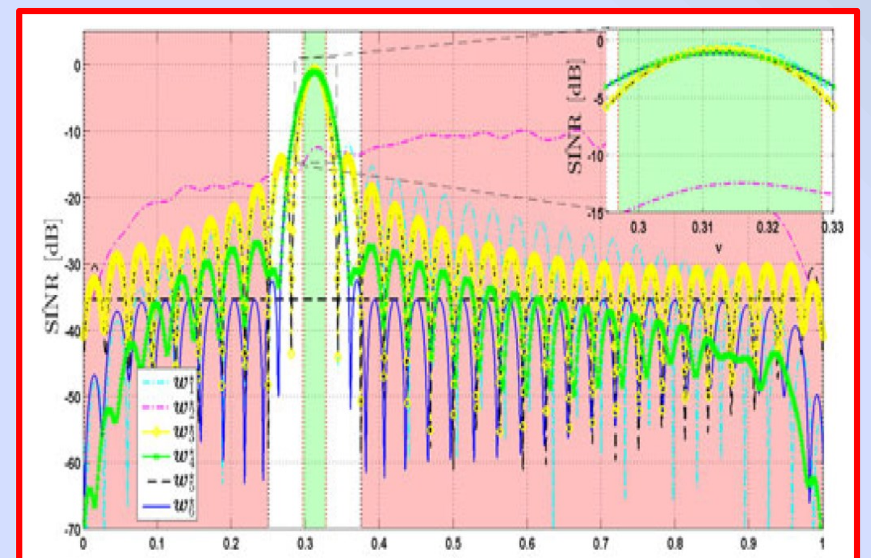
- $N = 32,\ K = N$
- $\mathrm{MC} = 100,\ \zeta = -32$ dB
- $\sigma^2_{n,\mathrm{dB}} = 2$ dB, $f_S = 0.2$
- $\mathrm{CNR}_{S,\mathrm{dB}} = 10$ dB, $\mathrm{CNR}_{G,\mathrm{dB}} = 25$ dB
- $\rho_S = 0.8,\ \rho_G = 0.95$

$$\hat{\mathrm{SINR}}(\nu) = \frac{1}{\mathrm{MC}} \sum_{m=1}^{\mathrm{MC}} \frac{|\boldsymbol{w}^{m\dagger} \boldsymbol{s}(\nu)|^2}{\boldsymbol{w}^{m\dagger} \boldsymbol{R} \boldsymbol{w}^m}$$

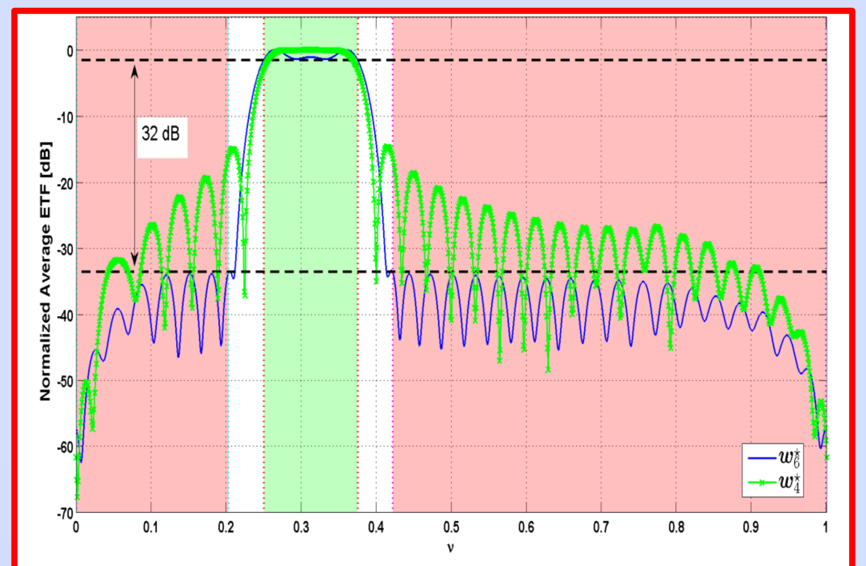
$$\boldsymbol{R}_0(n,m) = \mathrm{CNR}_S \rho_S^{(n-m)^2} e^{j2\pi(n-m)f_S} + \mathrm{CNR}_G \rho_G^{|n-m|} + \sigma_n^2 \delta_{n-m},$$



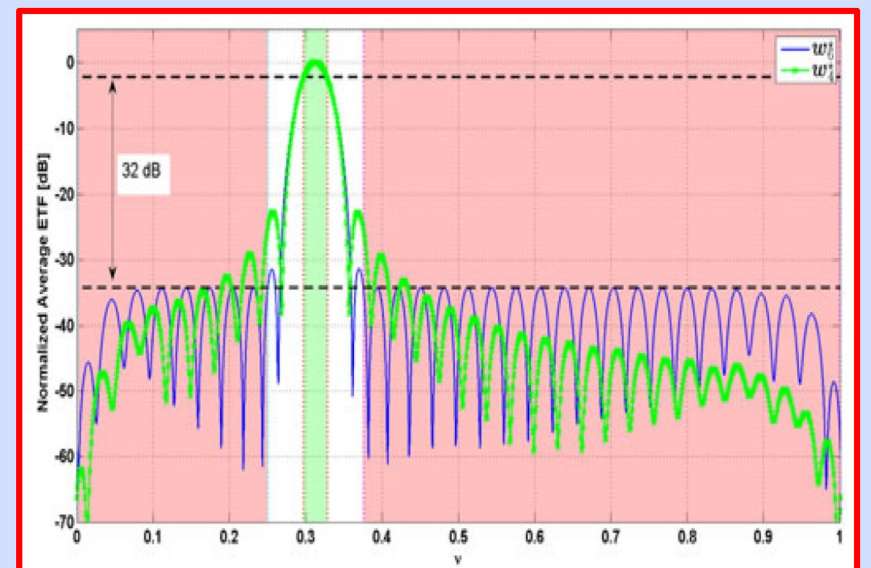
$\hat{\mathrm{SINR}}$ vs ν for $\boldsymbol{w}_i^\star$, $i = 1, \ldots, 6$. $\zeta = -32$ dB, $\Theta_1 = [\bar{\nu} - 2/N, \bar{\nu} + 2/N]$, $\Theta_2 = [0, \bar{\nu} - 3.5/N] \bigcup [\bar{\nu} + 3.5/N, 1]$, and $\bar{\nu} = 10/N$



$\hat{\mathrm{SINR}}$ vs ν for $\boldsymbol{w}_i^\star$, $i = 1, \ldots, 6$. $\zeta = -32$ dB, $\Theta_1 = [\bar{\nu} - 1/(2N), \bar{\nu} + 1/(2N)]$, $\Theta_2 = [0, \bar{\nu} - 2/N] \bigcup [\bar{\nu} + 2/N, 1]$, and $\bar{\nu} = 10/N$



Normalized average ETF vs ν for $\boldsymbol{w}_4^\star$ and $\boldsymbol{w}_6^\star$. $\zeta = -32$ dB, $\Theta_1 = [\bar{\nu} - 2/N, \bar{\nu} + 2/N]$, $\Theta_2 = [0, \bar{\nu} - 3.5/N] \bigcup [\bar{\nu} + 3.5/N, 1]$, and $\bar{\nu} = 10/N$



Normalized average ETF vs ν for $\boldsymbol{w}_4^\star$ and $\boldsymbol{w}_6^\star$. $\zeta = -32$ dB, $\Theta_1 = [\bar{\nu} - 1/(2N), \bar{\nu} + 1/(2N)]$, $\Theta_2 = [0, \bar{\nu} - 2/N] \bigcup [\bar{\nu} + 2/N, 1]$, and $\bar{\nu} = 10/N$

SECTION V: CONCLUSIONS

- **The constrained design of robust radar Doppler filters has been considered, accounting for**:
  - The **possible mismatches between the design and the operative conditions**;
  - The **control** of the **filter sidelobe behavior**;
  - The **output SINR** as performance measure.
- The developed filtering strategy is able to **mitigate the deleterious effects of modeling mismatches providing satisfactory worst-case SINR values**. This is **crucial in defense applications**.
- It could be interesting to:
  - **Perform a real radar data analysis;**
  - **Extend the framework to a STAP scenario.**