Fractional Fourier Transform based Sparse Underwater Channel Estimation

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1. Project objective

Design a hybrid sparse channel estimation system based on Fractional Fourier Transform (FrFT) for orthogonal frequency division multiplex (OFDM) scenario to exploit channel sparsity of underwater acoustic (UWA) channel. A novel channel dictionary matrix based on chirp signals is constructed and mutual coherence is adopted to evaluate its preservation of sparse information. In addition, Compressive Sampling Matching Pursuit (CoSaMP) is implemented to estimate the sparse channel coefficients.

2. Background and Motivation

- Underwater acoustic communication (UWA) suffers from doubly selective (both time and frequency) channel, attributed to long delay spread and serious Doppler spread from high mobility between transmitter and receiver.
- The Character of channel in Figure 1 is sparse. The majority of channel energy is concentrated on a few paths. Therefore, the compressed sensing (CS) can be applied to estimate the channel coefficients.

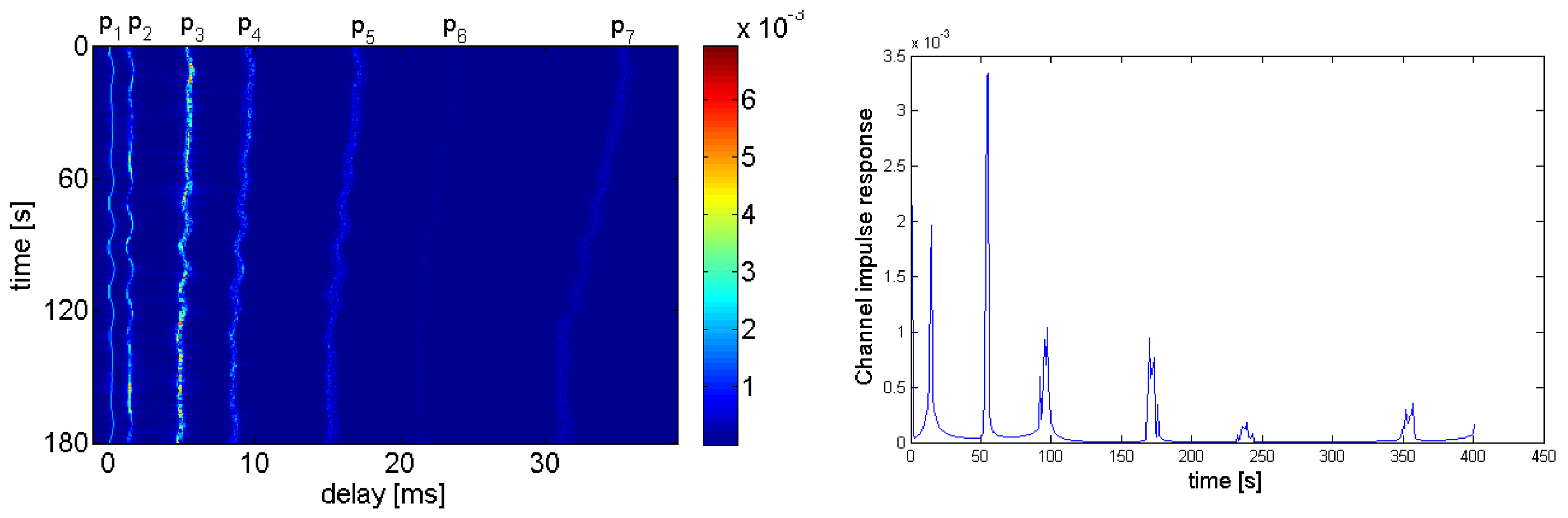

Figure 1: Impulse response of underwater acoustic channel

3. Proposed approach

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3-1 FrFT-Transceiver

The proposed Fractional fourier based channel estimation is called (FrFT-CS) due to implementation of compress sensing (CS), and FrFT based transceiver is shown in Figure 2. The input signal**s**, modulated by inverse FrFT (IFrFT), goes through the UWA channel with additive Gaussian noise $n(t)$ after appending cyclic prefix. The corrupted received signal $y(t)$, demodulated by FrFT, is processed by sparse channel estimation, including FrFT based dictionary matrix which consists of delay and Doppler representative sets and CoSaMP optimization. The estimated channel information $\widehat{\mathbf{H}}$ is sent to following equalization and detection blocks.

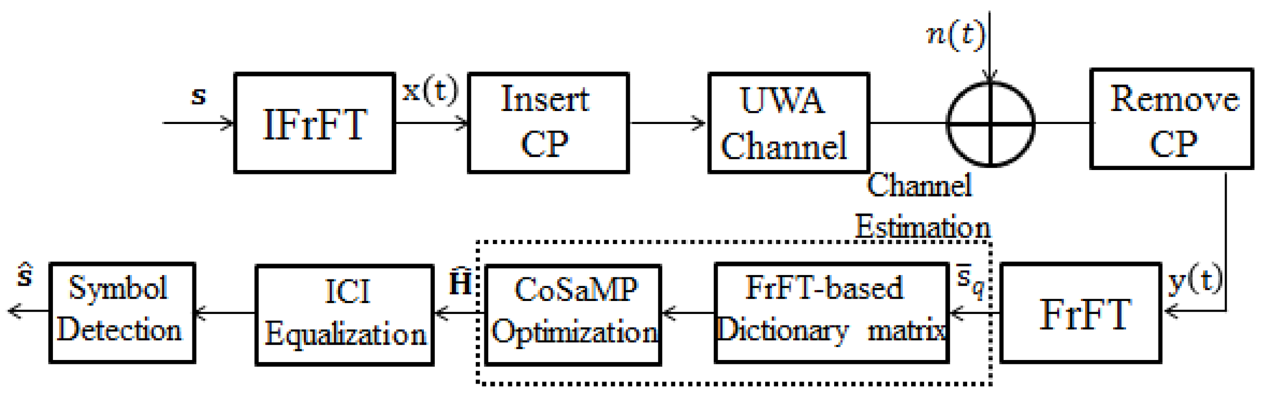

Figure 2: Diagram of FrFT-Transceiver

3-2 FrFT Dictionary Matrix Construction

The channel frequency response is denoted as

$$[\mathbf{H}]_{m,k} = \frac{1}{T}\left(\frac{\pi^2 sin\alpha^2}{T^2}\exp(k^2-m^2)\,cot\alpha\right)\sum_{i=1}^{L}\exp\left(-j\left(\frac{2\pi k\tau_i}{T}+\frac{\tau_i^2 cot\alpha}{2}\right)\right)\int_0^T h_i(t)e^{j\left(\frac{2\pi(k-m)}{T}+\tau_i cot\alpha-2\pi f_i\right)t}dt$$

In which τ_i, f_i and denote the delay, the Doppler shift of the i-th path respectively. $0<\alpha<\pi$ is the angle of rotation of FrFT between time and frequency domain.

The channel frequency response can be separated into three parts

$$\mathbf{H}=\sum_{i=1}^{L}\boldsymbol{\varepsilon}_i\boldsymbol{\rho}_i\boldsymbol{\varphi}_i$$

$$[\boldsymbol{\varphi}_i]_{m,k}=\int_0^T e^{j\left(\frac{2\pi(k-m)}{T}+\tau_i cot\alpha-2\pi f_i\right)t}dt=\mathrm{sinc}([\boldsymbol{\beta}]_{m,k}T)\exp(j\pi[\boldsymbol{\beta}]_{m,k}T)$$

$\boldsymbol{\varphi}_i$ is the contribution of ICI caused by Doppler frequency in $\boldsymbol{\beta}$.

$$[\boldsymbol{\beta}]_{m,k}=\frac{k-m}{T}+\frac{\tau_i cot\alpha}{2\pi}-f_i$$

$$[\boldsymbol{\rho}_i]_{m,m}=\exp\left(-j\left(\frac{2\pi k\tau_i}{T}+\frac{\tau_i^2 cot\alpha}{2}\right)\right)$$

$\boldsymbol{\rho}_i$ is a diagonal matrix the delay of i^{th} path among all the subcarriers and ε_i denotes the complex path gain.

Due to the sparse character of UWA channel, both the multipath delay and Doppler shift can be discretized to build the dictionary representation matrix, denoted as $\bar{\tau}$ and f_r respectively.

$$\bar{\tau}\epsilon\left\{0,\frac{T}{\vartheta B},\frac{2T}{\vartheta B},\dots T_g\right\}$$

$$f_r\epsilon\{-f_{max},-f_{max}+\Delta f,-b_{max}+2\Delta f,f_{max}\}$$

in which ϑ is the oversampling factor, and f_{max} is selected as residual Doppler shift after process by FrFT.

The received signal can be written as

$$\mathbf{Y}_p=\boldsymbol{Ah}+\boldsymbol{n_p}$$

$$=[\boldsymbol{\gamma}(\boldsymbol{f_1}),\dots\boldsymbol{\gamma}(\boldsymbol{f_{N_f}})]_{,}\begin{bmatrix}\varepsilon_1\\ \vdots\\ \varepsilon_{N_f N_\tau}\end{bmatrix}+\boldsymbol{n_p}$$

where $\boldsymbol{\gamma}(\boldsymbol{f_1})=[\boldsymbol{\rho_1\varphi_1 s}\ \dots\dots\ \boldsymbol{\rho}_{N_\tau}\boldsymbol{\varphi}_{N_\tau}\mathbf{s}]$, $\boldsymbol{\varphi_1}$ is a the number of columns of the measurement matrix is much greater than that of the row, namely $N_f N_\tau \gg L$.

3-2 Mutual coherence of FrFT Dictionary Matrix

The Dictionary matrix **A** should meet the requirements of restricted isometry property (RIP), and another method which measures the mutual coherence of the dictionary matrix is expressed as

$$\mu(\mathbf{A})=\max_{1\le i,j\le,i\ne j}\frac{|\langle\boldsymbol{a_i a_j}\rangle|}{\|\boldsymbol{a_i}\|\|\boldsymbol{a_j}\|}$$

where $\boldsymbol{a_i}$ and $\boldsymbol{a_j}$ represent the i^{th} and j^{th} column of **A** respectively.

3-3 Compressive Sampling Matching Pursuit (CoSaMP) Algorithm

CoSaMP is implemented to search the optimal result of channel coefficients.

Step1: The residual component $\boldsymbol{r_l}$, and iteration number $\boldsymbol{l}$ is initialized as $\boldsymbol{r_0}=\mathbf{Y}_p$ and $\boldsymbol{l}=\mathbf{1}$. Both the candidate support $\boldsymbol{C_t}$ and indices set $\boldsymbol{D_t}$ are initialized as empty sets, denoted as $\boldsymbol{D_0}=\emptyset$, $\boldsymbol{C_0}=\emptyset$ respectively.

Step2: The columns of dictionary matrix $\boldsymbol{A}$ is correlated with $\boldsymbol{r}$ one by one**,** expressed as $\boldsymbol{u}=\boldsymbol{A^H r_{l-1}}$, and the 2K largest elements in $\boldsymbol{u}$ are chosen by magnitude and added them to candidate support**,** as $\boldsymbol{C_t}=\boldsymbol{C_{t-1}}\cup\boldsymbol{a_j}$ where $\boldsymbol{a_j}$ is the j^{th} column of $\boldsymbol{A}$**.** The column index $\boldsymbol{J_l}$ are inserted to indices set, $\boldsymbol{D_l}=\boldsymbol{D_{l-1}}\cup\boldsymbol{J_l}$

Step3: Solve the Least Square (LS) Problem:$\boldsymbol{\varepsilon_l}=(\boldsymbol{C_l}^T\boldsymbol{C_l})^{-1}\boldsymbol{C_l}^T\mathbf{Y}_p$.

Step4: The K largest absolute elements are chosen from $\boldsymbol{\varepsilon_l}$, denoted as $\boldsymbol{\varepsilon_{tK}}$, and they are corresponded to the K columns in dictionary matrix $\boldsymbol{A}$, denoted as $\boldsymbol{C_{tK}}$.

Step5: The residual component is updated as $\boldsymbol{r_{temp}}=\mathbf{Y_p}-\boldsymbol{C_{tK}}(\boldsymbol{C_{tK}}^T\boldsymbol{C_{tK}})^{-1}\boldsymbol{C_{tK}}^T\mathbf{Y_p}$.

The **Step2** to **Step5** are repeated until the iteration number $\boldsymbol{l}$ reached the known sparsity or the norm of $\|\boldsymbol{r}\|^2$ is reduced below a predetermined threshold.

4. Simulation results

- **Parameter of OFDM**
  - -Subcarriers number: 256
  - -Pilot number: 64 (randomly distributed)
  - -Duration: 104.5ms
  - -Guard Interval：24.6ms
  - -Baseband frequency bandwidth: 9.77 kHZ
  - -Baseband sampling frequency
  - -Pilot is randomly distributed
- **Parameter of UWA channel**
  - -Path number: 7
  - -Multipath delay: exponentially distributed with mean $\mathrm{E}[\tau_{p+1}-\tau_p]=1ms$
  - -Doppler spread: $f_d=0.10\Delta f$
  - -Average power reduce exponentially with delay, and the amplitude is Rayleign distributed
  - -Signal to Noise Ratio (SNR): 0 to 30dB

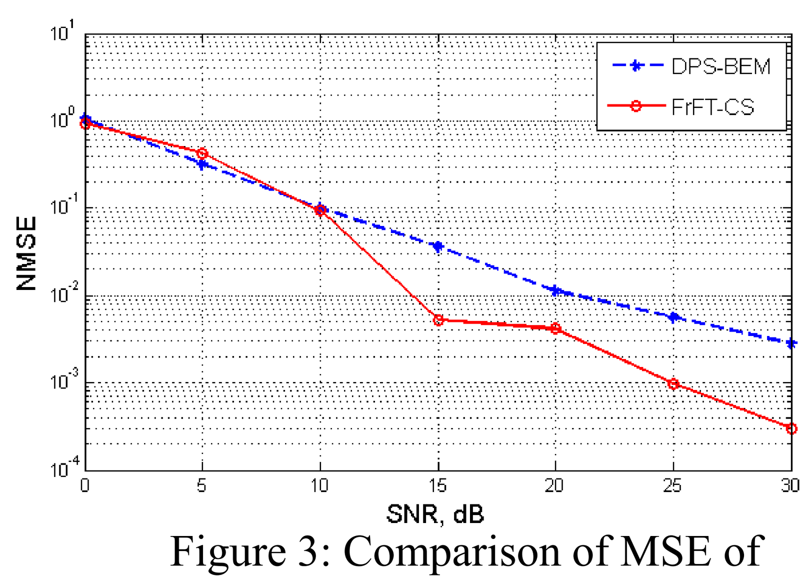

Figure 3: Comparison of MSE of DPS-BEM and FrFT-CS

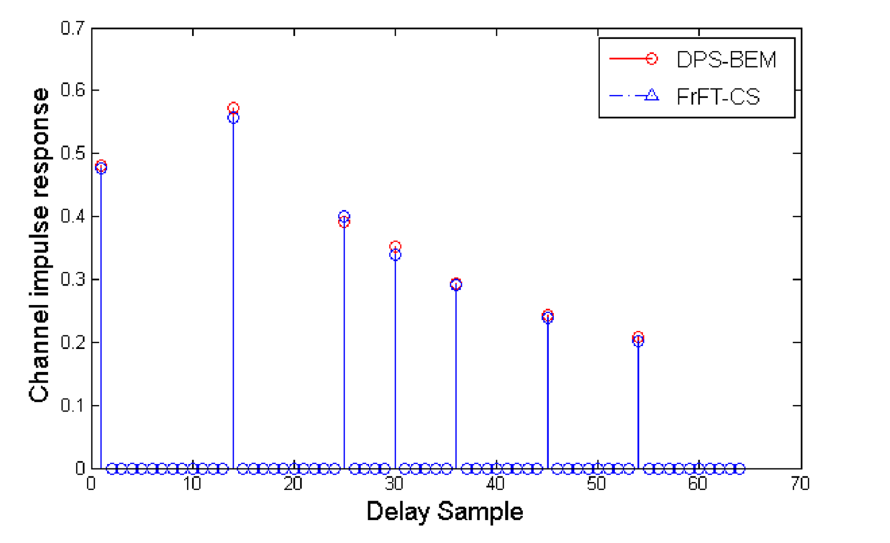

Figure 4: Comparison of channel impulse response estimated

5. Future plans

- Further work attempt to improve the suitability of larger motion-induced Doppler range and FrFT optimal order, and the synchronization, including joint Doppler and Delay estimation based on ambiguity function with FrFT modulation.
- Future work focus on implementation of proposed algorithm to real UWA experiment.

[1] Y. Chen, "Partial Fractional Fourier Transform (PFrFT)-OFDM for underwater acoustic communication," *European Signal Processing Conference (EUSIPCO)*, Nice, France, 2015.

[2] Y. Chen, "Fractional Cosine Transform (FrCT)-Turbo based OFDM for underwater acoustic communication." *Sensor Signal Processing for* Defence *(SSPD)*, Edinburgh, Britain, 2015.

[3] B. Li, S. Zhou. M. Stojanovic, L. Freitag, and P. Willet, "Multicarrier Communication over underwater acoustic channels with non-uniform Doppler shifts," *IEEE J. Ocean. Eng.*, vol. 33, no. 2, pp. 198-209, Apr. 2008.