Joint Radar Waveform and Bank of Filter Design for Wind Farm Clutter Mitigation

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Motivation and Aim

- The presence of wind farms within the surveillance area of a radar system affects dramatically its target detection capabilities. Indeed, the *large radar cross-section* of wind turbines, and the *time-varying Doppler modulation* injected on the radar returns decreases the capability of a radar to detect moving targets.
- **Aim** Development of a novel strategy for the joint design of radar waveform and bank of filters in order to maximize the detection probability of a distributed radar network.

Multi-Static Radar System

A multi-static radar system is considered, with one transmitter and Q receivers:

- the receivers are widely spaced: the diversity in aspect angle with the wind turbines, generates different Doppler signatures at each receiver.
- the surveillance region is divided into K cells, of dimensions driven by the resolution of the system itself.
- each cell contains either a transmitter/receiver node or a wind turbine.

Figure 1 shows an example scenario, with a transmitter/receiver node of the network located at the radar cell $k = 1$ and another receiver in $k = 10$. Two wind turbines are placed in $k = 5$ and $k = 76$, respectively.

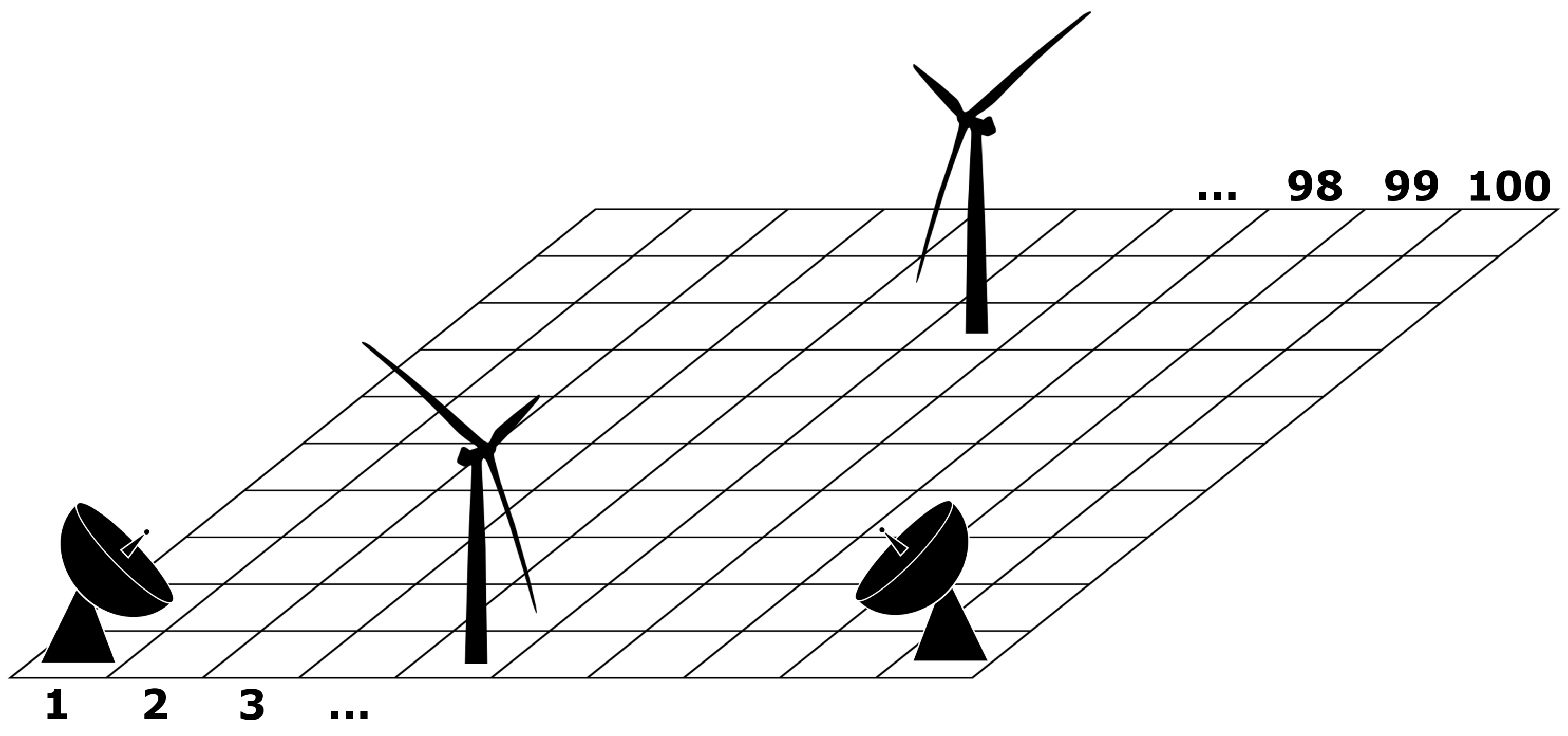

Figure 1: Example scenario.

The transmitted signal $\boldsymbol{s}$ is a slow-time sequence of N coded pulses:

$$\boldsymbol{s} = \left[s(1), \dots, s(N)\right]^T \in \mathbb{C}^N \tag{1}$$

Signal Model and Problem Formulation

The model of the signal received by the q-th receiver from the k-th radar cell $\boldsymbol{v}^{(q,k)}(\boldsymbol{s})$ consists of three terms:

$$\boldsymbol{v}^{(q,k)}(\boldsymbol{s}) = \boldsymbol{t}^{(q,k)}(\boldsymbol{s}) + \boldsymbol{d}^{(q,k)}(\boldsymbol{s}) + \boldsymbol{n}^{(q)} \tag{2}$$

- $\boldsymbol{t}^{(q,k)}(\boldsymbol{s})$ represents the return from the perspective target.
- $\boldsymbol{d}^{(q,k)}(\boldsymbol{s})$ contains the filtered signal-dependent interfering samples, due to clutter and rotation of the wind turbine's blade.
- $\boldsymbol{n}^{(q)}$ is the filtered signal-independent coloured noise.

The joint design of the radar code $\boldsymbol{s}$ and the filters:

$$\boldsymbol{w}_{k,i} = \left[\boldsymbol{w}_{k,i}^{(1)\dagger}, \dots, \boldsymbol{w}_{k,i}^{(Q)\dagger}\right], \quad i \in \mathcal{A}_k \tag{3}$$

is accomplished by maximising the worst-case Signal-to-Interference plus Noise Ratio (SINR), that is:

$$\max_{\boldsymbol{s},\{\boldsymbol{w}_{k,i}\}} \ \mathrm{SINR}_B \tag{4}$$

where

$$\mathrm{SINR}_B = \min_{k,i} \frac{\sum_{q=1}^{Q} \mathrm{var}\left(\alpha^{(q,k)}\right) \left| \left(\boldsymbol{s} \odot \boldsymbol{p}\left(\mu_i^{(q)}\right)\right)^\dagger \boldsymbol{w}_{k,i}^{(q)} \right|^2}{\boldsymbol{w}_{k,i}^\dagger \boldsymbol{\Sigma}_{\boldsymbol{d}}^{(k)}(\boldsymbol{s})\, \boldsymbol{w}_{k,i} + \boldsymbol{w}_{k,i}^\dagger \boldsymbol{\Sigma}_{\boldsymbol{n}}\, \boldsymbol{w}_{k,i}} \tag{5}$$

$\mathcal{A}_k$ is the set of velocity bins of interest for the k-th cell, where each velocity bin corresponds to a specific pull of Q values, $\left\{\mu_i^{(1)}, \dots, \mu_i^{(Q)}\right\}$. Moreover:

- var $\left(\alpha^{(q,k)}\right)$ is the power of the target return.
- $\boldsymbol{\Sigma}_{\boldsymbol{n}}$ is a block diagonal matrix formed by the coloured noise covariance matrices from all the receivers.

Signal Model and Problem Formulation (cont.)

- $\boldsymbol{\Sigma}_{\boldsymbol{d}}^{(k)}(\boldsymbol{s})$ is a block diagonal matrix; each sub-matrix is the covariance matrix of the filtered signal-dependent interferences $\boldsymbol{d}^{(q,k)}$ on varying $q = 1, \dots, Q$. This term embeds the information on the Doppler modulation generated by the wind turbines' blades.

Optimisation Problem

The optimisation problem is formulated as follows:

$$\mathcal{P} \begin{cases} \max\limits_{\boldsymbol{s},\{\boldsymbol{w}_{k,i}\}} \ \mathrm{SINR}_B & \\ \text{s.t.} & \|\boldsymbol{s}\|^2 = 1 \\ & \|\boldsymbol{s} - \boldsymbol{s}_0\|^2 \leqslant \tau \end{cases} \tag{6}$$

The first constraint rules the overall energy of the code; the second one controls some critical properties of the signal, forcing $\boldsymbol{s}$ to be similar (depending on the parameter $0 \leqslant \tau \leqslant 2$) to a prefixed code $\boldsymbol{s}_0$.

Because the objective function is non-convex, and $\|\boldsymbol{s}\|^2 = 1$ defines a non-convex set, the optimisation problem is non-convex.

However, it can be shown that, for any $0 \leqslant \tau < 2$, problem $\mathcal{P}$ is equivalent to the following:

$$\mathcal{P}_1 \begin{cases} \max\limits_{\boldsymbol{s},\{\boldsymbol{w}_{k,i}\}} \ \mathrm{SINR}_B & \\ \text{s.t.} & \|\boldsymbol{s}\|^2 \leqslant 1 \\ & \Re\left(\boldsymbol{s}_0^\dagger \boldsymbol{s}\right) \geqslant \tau_1 \end{cases} \tag{7}$$

where $\tau_1 = 1 - \frac{\tau}{2}$. Problem $\mathcal{P}_1$ is still non-convex but, differently from $\mathcal{P}$, it shares a convex feasible set. Following [1], a solution to $\mathcal{P}_1$ is proposed that consists in alternatively finding two variable blocks: one is the transmit signal $\boldsymbol{s}$, while the other is the set of all the receive filters $\{\boldsymbol{w}_{k,i}\}$. A closed form optimal solution for the bank of filters is provided, while the slow-time code $\boldsymbol{s}$ is obtained by exploiting some results from the Generalised Fractional Programming (GFP) theory [2],[3].

Results

The scenario used for the simulation is the one depicted in Figure 1; moreover:

- the wind turbines have 3 blades of length 45 m, rotating at 0.24 and 0.20 Rounds per Second (RPS), respectively, and they face south and south-east.
- the radar transmits a sequence of $N = 20$ phased coded pulses in L-band, with $PRI = 1/2$ ms, while the reference code $\boldsymbol{s}_0$ is a generalised Barker code.

The result of the simulation, reported in Figure 2 in terms of SINR_B over the number of iterations for two values of the similarity constraint τ, shows an improvement of the worst-case SINR of about 3.6 $d\mathrm{B}$.

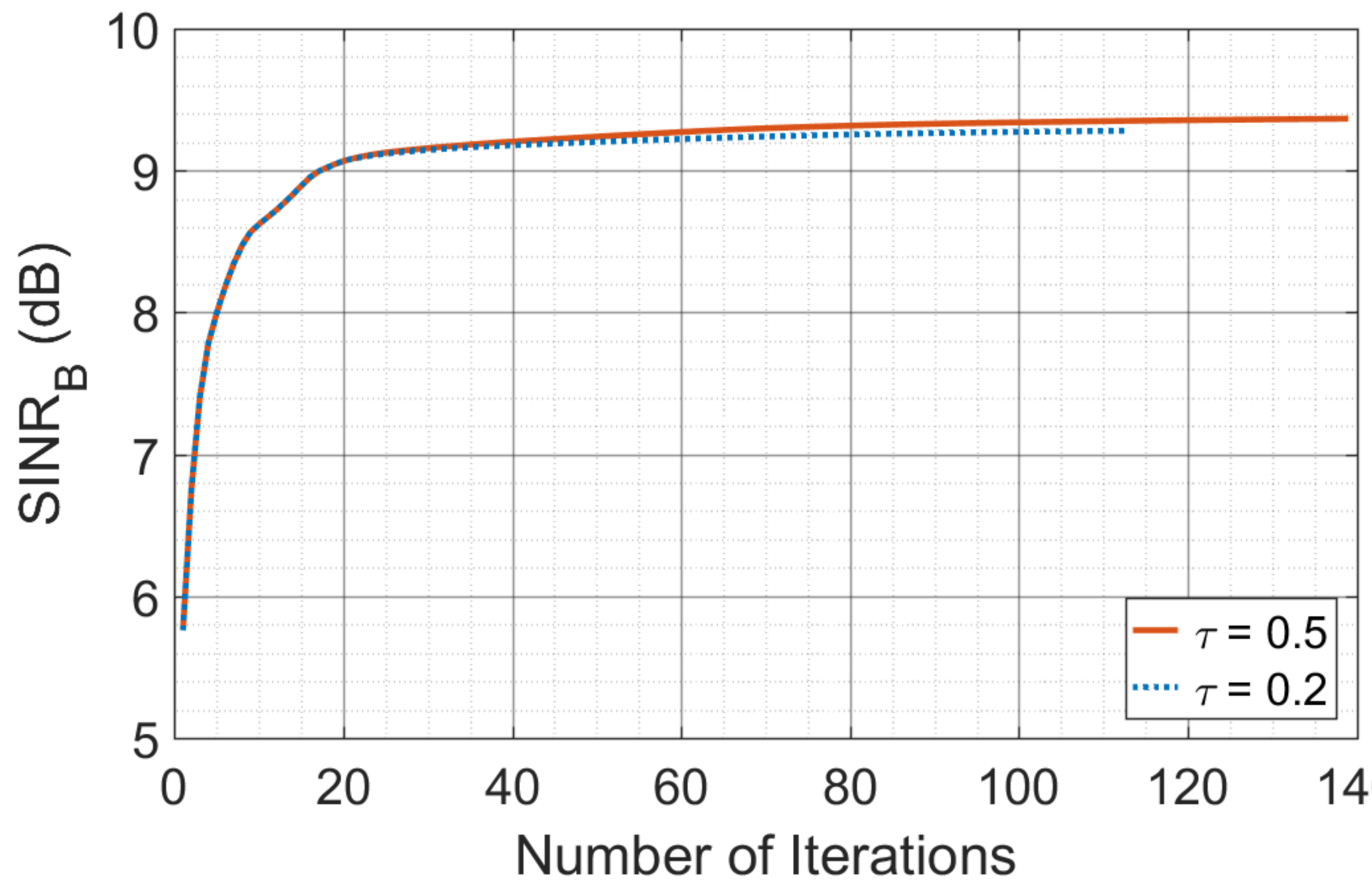

Figure 2: Analysis results: SINR_B monotonically increasing property of the proposed method.

Conclusions

The proposed framework allows to jointly adjust the slow-time radar waveform and the weights of the receiver filters aiming to optimize the SINR for a multi-static radar system in presence of wind turbine interference. The performance assessed on simulated data showed that the proposed method is effective in rejecting the interference from the wind farms, with an improvement of the worst-case SINR of about 3.6 $d\mathrm{B}$.

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