Second-order Statistics for Threat Assessment with the PHD Filter

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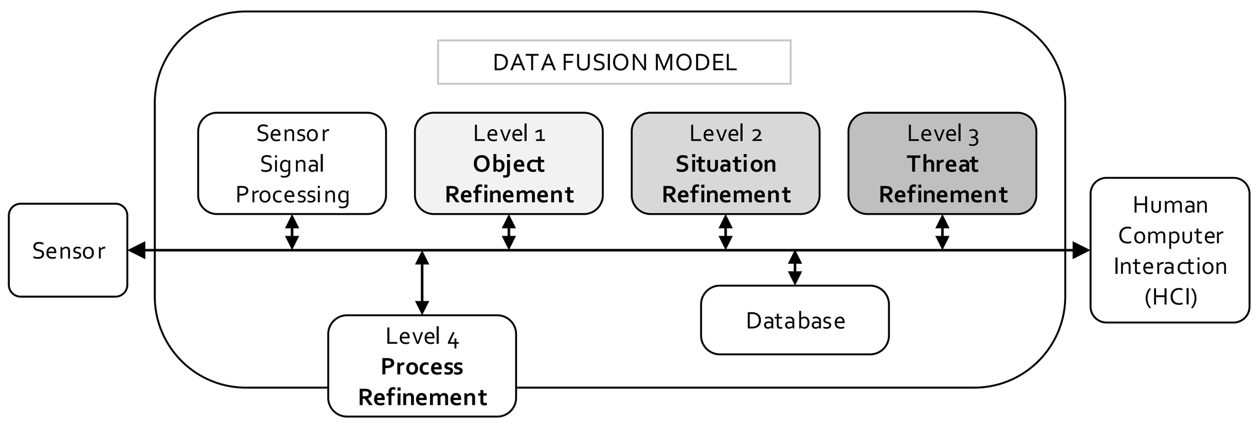
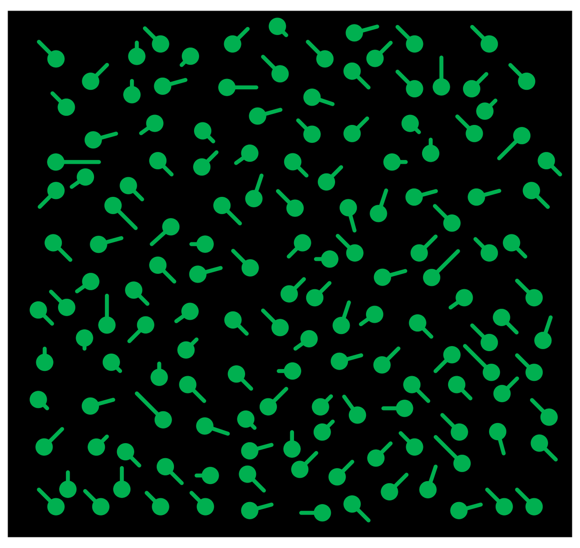
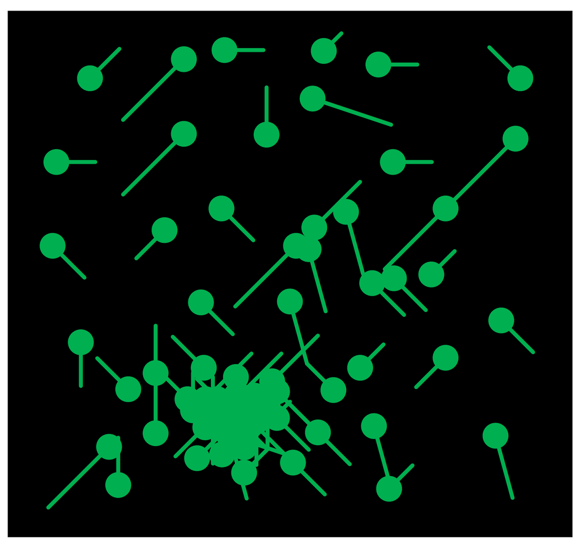
Introduction



– Threat assessment is a high-level data fusion process (Level 2/3).
– It concerns estimation and prediction of threats in the environment.
– Human operator is heavily involved in this process.
– Advances needed to aid operators and enable autonomous systems.

Description of the problem





– Multi-object scenarios can be particularly stressful for an operator.
– Describing a population automatically may improve the situation:
  (a) *what is the aggregated threat level of a population of objects?*
  (b) *what is the expected number of threatening objects?*, etc.
– Existing solutions: a point estimate without a quality indicator [2].
– Problem: reliability of an estimate cannot be established.
– **Objective: to obtain a measure of quality for an estimate.**

Probability Hypothesis Density filter

Operator's knowledge about the population is inferred from measurements and maintained by a multi-object filter. The PHD filter is an approximate filter that only propagates the Probability Hypothesis Density (PHD) describing the population, denoted by μ, and also called the density of the first-order factorial moment of the *point process* or *intensity function*. The filter's recursion at time step k consists of a *time prediction* step and an *data update* steps given by [2]

$$\mu_{k|k-1}(x) = \mu_k^{\mathrm{b}}(x) + \int_{\mathcal{X}} m_{k|k-1}(x|\bar{x}) p_{\mathrm{s},k}(\bar{x}) \mu_{k-1}(\bar{x}) \mathrm{d}\bar{x}, \quad (1)$$

$$\mu_k(x) = \mu_k^{\phi}(x) + \sum_{z \in Z_k} \frac{\mu_k^z(x)}{\mu_k^{\mathrm{fa}}(z) + \int_{\mathcal{X}} \mu_k^z(x) \mathrm{d}x}, \quad (2)$$

with the *missed detection* and *association* terms given by

$$\mu_k^{\phi}(x) = (1 - p_{\mathrm{d},k}(x)) \mu_{k|k-1}(x),$$
$$\mu_k^{z}(x) = p_{\mathrm{d},k}(x) g_k(z|x) \mu_{k|k-1}(x),$$

where $\mu_{k|k-1}(\cdot)$ and $\mu_k(\cdot)$ are, respectively, the predicted and updated intensity functions;
$\mu_k^{\mathrm{b}}(\cdot)$ and $\mu_k^{\mathrm{fa}}(\cdot)$ are, respectively, the intensity functions of newborn objects and false alarms;
$Z_{1:k}$ is the sequence of multi-object observations collected by time k, where Z_k is a set of single-object measurements collected at time k;
$g_k(\cdot|\cdot)$ is the single-object measurement likelihood;
$p_{\mathrm{s},k}(\cdot)$ and $p_{\mathrm{d},k}(\cdot)$ are, respectively, the probability of an object survival and its probability of detection;
$m_{k|k-1}(\cdot|\cdot)$ is the single-object Markov transition kernel, describing the time evolution of an object.

Mean and variance of the cumulative threat level

The aggregated threat of a population of objects with states $x_{1:n}$ is described by its *cumulative threat level*

$$\mathcal{T}(x_{1:n}) = \sum_{1 \le i \le n} \tau(x_i), \quad (3)$$

where τ is a function $\tau : \mathcal{X} \to [0, 1]$ evaluating the threat level of an individual with state x. As a consequence, the threat level of the observed population will be described by a "regular" real-valued random variable T, its statistics. Note that determination of the second-order statistics – of variance here and correlation in [3], – is enabled by original representation of a population as a point process.

Theorem 1 (Mean cumulative threat level [2]).
Under the assumptions of the PHD filter and considering cumulative threat as in (3)*, the first-order raw moment or mean of the cumulative threat level of the updated process, cf.* (2)*, at time step k is given by*

$$\mathbb{E}[\mathrm{T}_k] = \int_{\mathcal{X}} \tau_k(x) \mu_k^{\phi}(x) \mathrm{d}x + \sum_{z \in Z_k} \frac{\int_{\mathcal{X}} \tau_k(x) \mu_k^z(x) \mathrm{d}x}{\mu_k^{\mathrm{fa}}(z) + \int_{\mathcal{X}} \mu_k^z(x) \mathrm{d}x}. \quad (4)$$

Theorem 2 (Variance in cumulative threat level – **Estimate's quality measure [main result]**).
Under the assumptions of the PHD filter and considering cumulative threat as in (3)*, the second-order central moment or variance in the cumulative threat level of the updated process, cf.* (2)*, at time step k is given by*

$$\mathrm{var}[\mathrm{T}_k] = \int_{\mathcal{X}} \tau_k^2(x) \mu_k^{\phi}(x) \mathrm{d}x + \sum_{z \in Z_k} \left[\frac{\int_{\mathcal{X}} \tau_k^2(x) \mu_k^z(x) \mathrm{d}x}{\mu_k^{\mathrm{fa}}(z) + \int_{\mathcal{X}} \mu_k^z(x) \mathrm{d}x} - \left(\frac{\int_{\mathcal{X}} \tau_k(x) \mu_k^z(x) \mathrm{d}x}{\mu_k^{\mathrm{fa}}(z) + \int_{\mathcal{X}} \mu_k^z(x) \mathrm{d}x} \right)^2 \right]. \quad (5)$$

Regional variance [1]. When interest lies in a specific region $B \subset \mathcal{X}$ the function τ can be selected to be the indicator function 1_B defined such that $1_B(x) = 1$ if $x \in B$, $1_B(x) = 0$ otherwise. The cumulative threat level statistics then reduce to the *regional statistics* describing the number of objects in B.

Simulated example

The threat level of x is evaluated w.r.t. to a point of interest $x_{\mathrm{o}} \in \mathcal{X}$ and a region of interest $B \subset \mathcal{X}$ by

$$\tau(x) = 1_B(x) \exp\left(-\frac{d(x, x_{\mathrm{o}})}{\alpha} - \frac{b^2(x, x_{\mathrm{o}})}{2\beta^2} \right), \quad (6)$$

where $1_B(x)$ evaluates whether an object with state $x = [\mathrm{x}, \mathrm{y}, \dot{\mathrm{x}}, \dot{\mathrm{y}}]^T$ belongs to the region B; the distance $d(x, x_{\mathrm{o}}) = \sqrt{(\mathrm{x} - \mathrm{x_o})^2 + (\mathrm{y} - \mathrm{y_o})^2}$ between the object x and the origin x_{o} is related to the object's capability to inflict negative effect; the object's direction $b(x, x_{\mathrm{o}}) = |\mathrm{atan2}(\dot{\mathrm{y}}, \dot{\mathrm{x}}) - \mathrm{atan2}(\mathrm{x_o} - \mathrm{x}, \mathrm{y_o} - \mathrm{y})|$ w.r.t. the point is related to object's intention to act hostile, where $\mathrm{atan2}(\mathrm{y}, \mathrm{x})$ is the four-quadrant inverse tangent function; α and β are positive-valued scaling parameters, here $\alpha = 2000\,\mathrm{m}$ and $\beta = 0.5$.

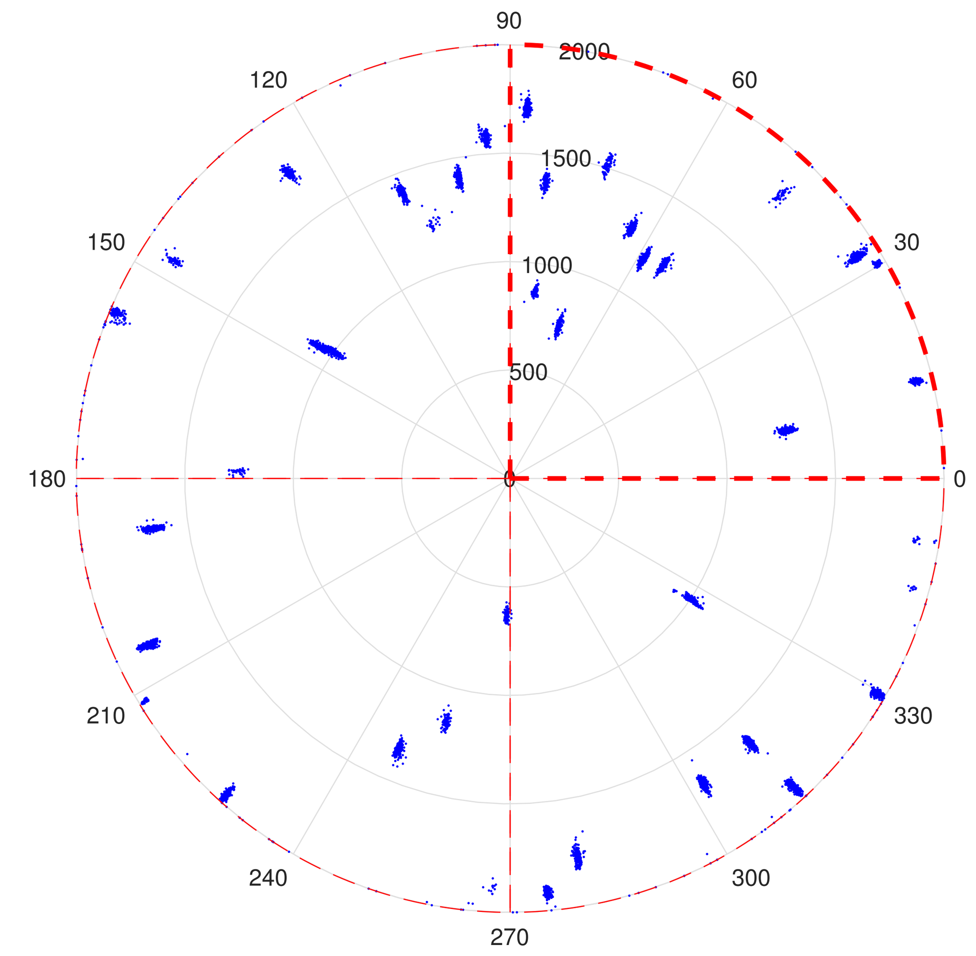


Fig. 1: Updated intensity μ_k in an SMC-PHD filter. The particles (in blue) are projected on the subspace of position variables. Regions of interest are depicted with red dashed lines and numbered counter-clockwise with the first region plotted with a thicker line. The sensor with state x_{o} is located at the origin.

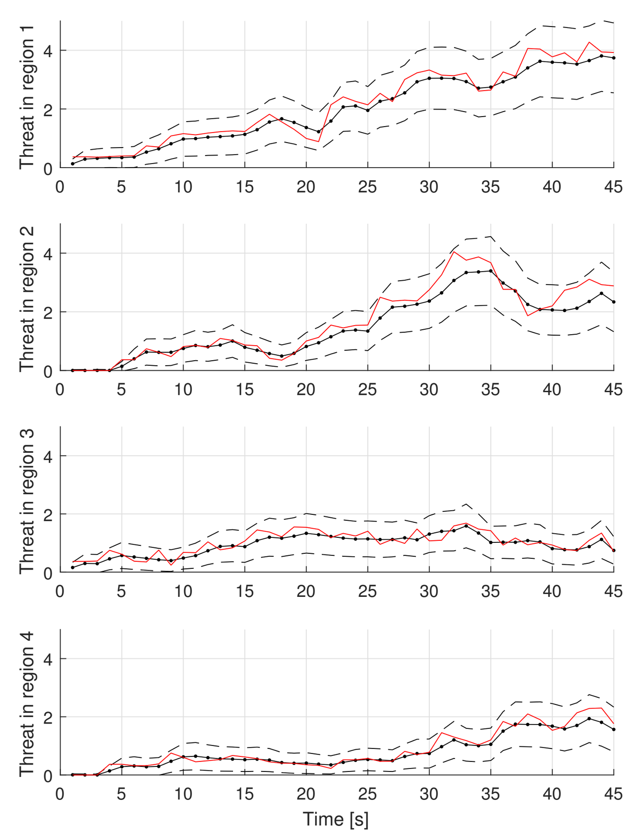


Fig. 2: Mean cumulative threat level and ± 1 standard deviation (square root of the variance, **which is the sought after quality measure**) in regions from 1 to 4. The ground truth value of the threat level is plotted with plain red line. The results are averages over 60 Monte Carlo runs.

Conclusions

- This work explores the problem of estimating a population's aggregated threat level from sensed data.
- It provides explicit expressions for the threat level statistics using quantities available from the PHD filter.
- The future work will be concerned with obtaining expressions for the *second-order* PHD filter [3], exploring alternative aggregations (e.g. multiplication) and exploiting second-order statistics for sensor management.

[dstl] EPSRC Engineering and Physical Sciences Research Council

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