Optimality Criteria for Adaptive Waveform Design in MIMO Radar Systems

SSPD 06/12/2017

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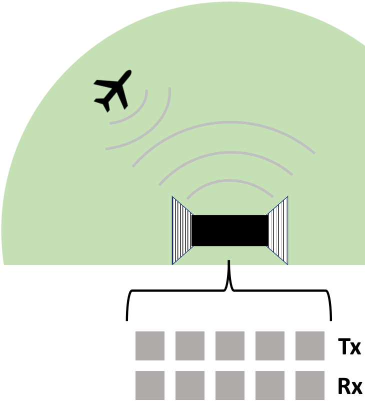
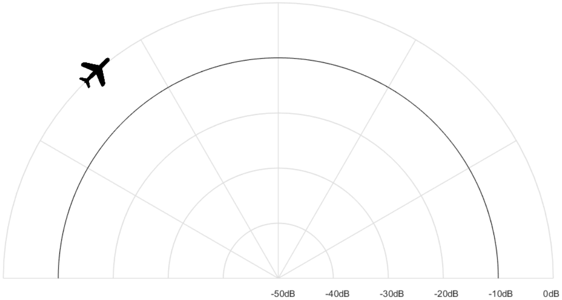
Presentation overview

- MIMO active sensing model
- Adaptive waveform design (AWD):
  - Problem statement
  - Our solution
- Assessing various optimisation criteria
- Conclusions

MIMO active sensing: radar example



AWD: the essential idea



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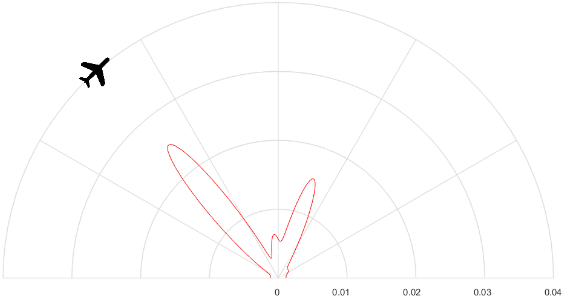

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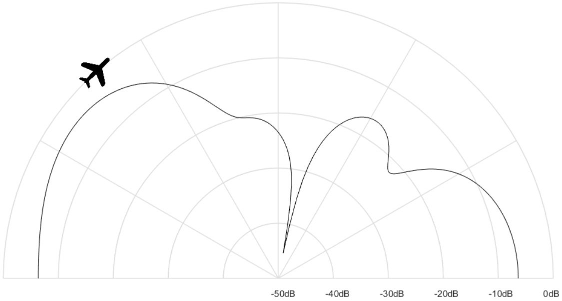

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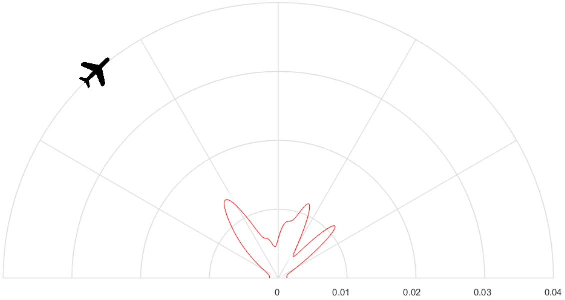

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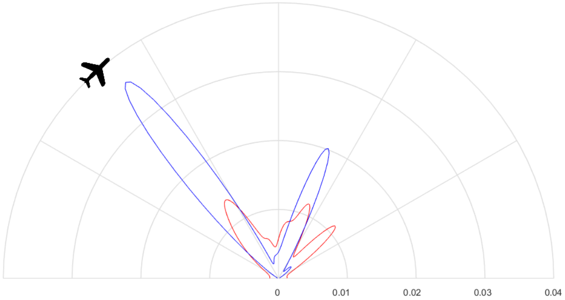

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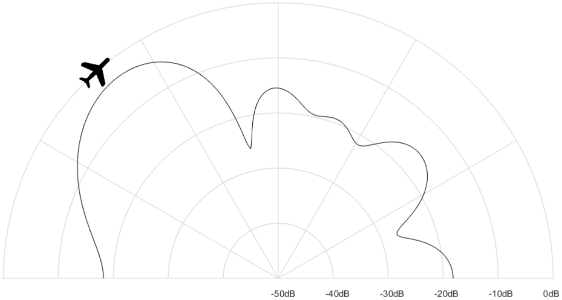


MIMO active sensing: system model

MIMO active sensing systems can be represented algebraically

$$\mathbf{X}_k = \mathbf{H}(\boldsymbol{\theta})\mathbf{S}_k + \mathbf{N}_k,$$

where $\boldsymbol{\theta} = [\phi; \Re(\alpha); \Im(\alpha)]$, and:

$$\mathbf{H} = \alpha \mathbf{a}_R(\phi)\mathbf{a}_T^T(\phi)$$

MMSE AWD: problem statement

$$\text{minimise: } \Sigma_k = \mathrm{tr}\left\{\mathbb{E}((\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta})(\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta})^T | \mathbf{X}^{k-1})\right\} \quad \text{wrt } \mathbf{S}_k,$$

$$\text{subj. to: } \mathrm{tr}\left\{\frac{1}{L}\mathbf{S}_k\mathbf{S}_k^H\right\} \leq \mathrm{P},$$

where

$$\hat{\boldsymbol{\theta}}_k = \mathbb{E}(\boldsymbol{\theta}|\mathbf{X}^k, \mathbf{S}^k)$$

'*it is difficult to obtain an analytic expression*' (Huleihel *et al* 2013)

MMSE AWD: our analytic solution

By definition

$$\Sigma_k = \iint (\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta})^T (\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta}) p(\hat{\boldsymbol{\theta}}_k, \boldsymbol{\theta} | \mathbf{X}^{k-1}, \mathbf{S}^k) \, \mathrm{d}\hat{\boldsymbol{\theta}}_k \, \mathrm{d}\boldsymbol{\theta},$$

which we show can be rearranged

$$\Sigma_k = \iint (\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta})^T (\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta}) p(\boldsymbol{\theta} | \mathbf{X}^{k-1}, \mathbf{S}^{k-1}) p(\mathbf{X}_k | \boldsymbol{\theta}, \mathbf{S}_k) \, \mathrm{d}\mathbf{X}_k \, \mathrm{d}\boldsymbol{\theta}$$

MMSE AWD: our numerical solution

$\boldsymbol{\theta}$ estimated using a particle filter, which yields:

$$\Sigma_k \approx \Sigma'_k = \int \sum_{i=1}^{N_P} \left(\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta}_k^{(i)}\right)^T \left(\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta}_k^{(i)}\right) w_k^{(i)} p(\mathbf{X}_k | \boldsymbol{\theta}_k^{(i)}, \mathbf{S}_k) \, \mathrm{d}\mathbf{X}_k$$

where

$$\hat{\boldsymbol{\theta}}_k \approx \frac{\sum_{i=1}^{N_P} w_k^{(i)} p(\mathbf{X}_k | \boldsymbol{\theta}_k^{(i)}, \mathbf{S}_k) \boldsymbol{\theta}_k^{(i)}}{\sum_{i=1}^{N_P} w_k^{(i)} p(\mathbf{X}_k | \boldsymbol{\theta}_k^{(i)}, \mathbf{S}_k)}$$

... but we still have an integral and a sum

MMSE AWD: our numerical solution (cont.)

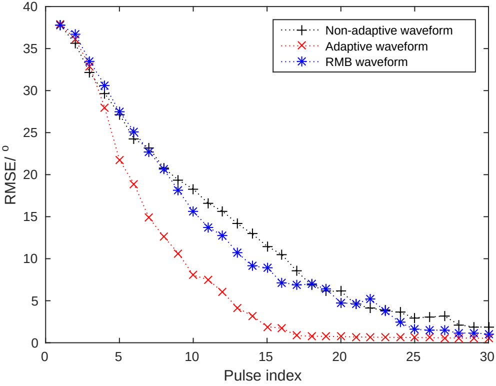
Solution, define a sample over the particles to do both sums in one go, for the mth sample:

$$\begin{aligned} \boldsymbol{\theta}_k'^{(m)} &\sim \sum_{i=1}^{N_P} w_k^{(i)} \delta(\boldsymbol{\theta}_k'^{(m)} - \boldsymbol{\theta}_k^{(i)}) \\ \mathbf{X}_k^{(m)} &\sim p(\mathbf{X}_k^{(m)} | \boldsymbol{\theta}_k'^{(m)}, \mathbf{S}_k(0)) \end{aligned}$$

which leads to our final approximate cost function expression

$$\Sigma_k' \approx \Sigma_k'' = \sum_{m=1}^{N_S} \frac{p(\mathbf{X}_k^{(m)} | \boldsymbol{\theta}_k'^{(m)}, \mathbf{S}_k) / p(\mathbf{X}_k^{(m)} | \boldsymbol{\theta}_k'^{(m)}, \mathbf{S}_k(0))}{\sum_{m'=1}^{N_S} p(\mathbf{X}_k^{(m')} | \boldsymbol{\theta}_k'^{(m')}, \mathbf{S}_k) / p(\mathbf{X}_k^{(m')} | \boldsymbol{\theta}_k'^{(m')}, \mathbf{S}_k(0))} \times \left(\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta}_k'^{(m)} \right)^T \left(\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta}_k'^{(m)} \right)$$

MMSE AWD: results



AWD video

Play: video2.avi

Expression of the expected covariance matrix

We can simply rearrange our MMSE solution to express the expected covariance matrix:

$$\Gamma_k = \iint (\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta}_k)(\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta}_k)^T p(\boldsymbol{\theta}_k | \mathbf{X}^{k-1}, \mathbf{S}^{k-1}) p(\mathbf{X}_k | \boldsymbol{\theta}_k, \mathbf{S}_k) \, \mathrm{d}\mathbf{X}_k \, \mathrm{d}\boldsymbol{\theta}_k,$$

this enables us to consider general optimality criteria:

$$\begin{aligned} &\text{minimise:} \quad \mathcal{M}(\Gamma_k) \qquad\qquad \text{w.r.t.} \mathbf{S}_k \\ &\text{subj. to:} \quad \mathrm{tr}\left\{\frac{1}{L}\mathbf{S}_k\mathbf{S}_k^H\right\} \leq \mathrm{P} \end{aligned}$$

Optimality criteria we consider

- A-optimal:

$$\mathcal{M}(\Gamma_k) = \mathrm{tr}(\Gamma_k)$$

- D-optimal:

$$\mathcal{M}(\Gamma_k) = \det(\Gamma_k)$$

- E-optimal:

$$\mathcal{M}(\Gamma_k) = \max\{\mathrm{Eig.}(\Gamma_k)\}$$

Simple interpretation of optimality criteria

For the case where the uncertainties of the locations of the various targets are independent:

$$\text{A-optimal:} \qquad \min \left(\sum_{i=1}^{Q} \sigma_i^2 \right),$$

$$\text{D-optimal:} \qquad \min \left(\prod_{i=1}^{Q} \sigma_i^2 \right),$$

$$\text{E-optimal:} \qquad \min \left(\max_{i=1:Q} \left(\sigma_i^2 \right) \right),$$

where σ_i^2 is the variance of the estimate of the location of the ith target.

Numerical comparison of optimality criteria

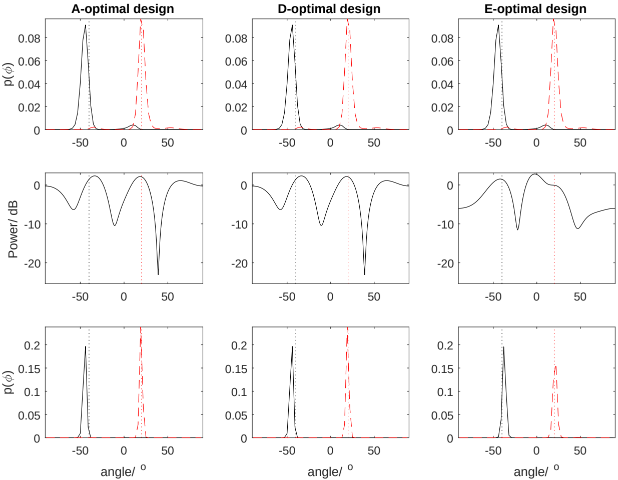
- A, D and E optimality are only distinct for multiple targets, thus we consider estimation of two targets
- For simplicity and consistency we perform exhaustive search over a pre-defined set of candidate waveforms for all criteria
- We consider two scenarios:
  - Two targets have comparable prior PDF
  - One target is more accurately estimated *a priori*

Numerical results for first scenario

Prior

Waveform

Posterior

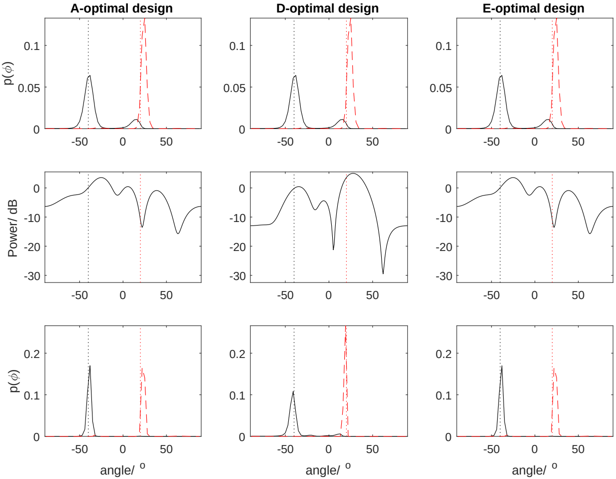


Numerical results for second scenario

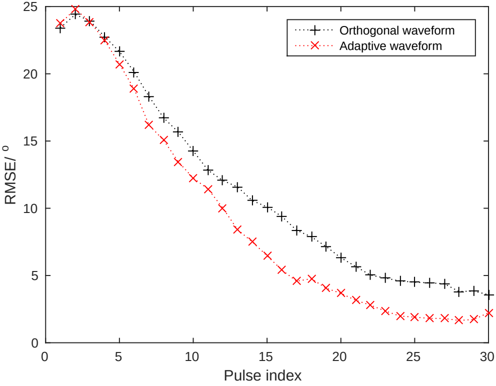
Prior

Waveform

Posterior



MMSE AWD with two targets



Conclusions

- Our MMSE AWD solution enables us to take a broader look at optimality criteria for AWD
- A-optimal (MMSE) AWD seems to be an all round good fit
- An actual system could be implemented which switches to E-optimal AWD if there is a particularly uncertain target which is of interest

References

- S. Herbert, J. Hopgood, and B. Mulgrew, *MMSE adaptive waveform design for active sensing with applications to MIMO radar*, to appear in IEEE transactions on signal processing 2017
- S. Herbert, J. Hopgood, and B. Mulgrew, *Optimality criteria for adaptive waveform design in MIMO radar systems*, SSPD 2017
- W. Huleihel, J. Tabrikian, and R. Shavit, *Optimal adaptive waveform design for cognitive MIMO radar*, IEEE transactions on signal processing, 2013