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# CORRELATION BASED CLASSIFICATION OF COMPLEX PRI MODULATION TYPES

Fotios Katsilieris\*, Sabine Apfeld, Alexander Charlish

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# Fraunhofer

## FKIE

**Sensor Data and Information Fusion**

Fraunhofer Institute for Communication, Information Processing and Ergonomics (FKIE)  
Wachtberg, Germany

\*Since April 2017 Fotios is with Airbus Defence and Space GmbH

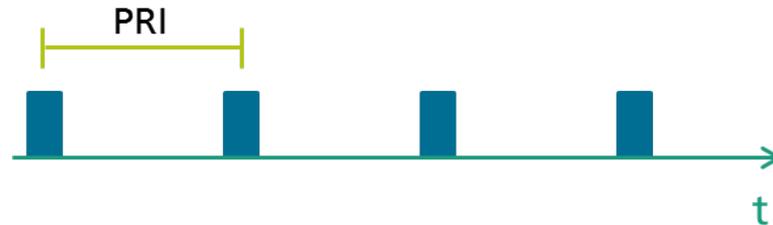
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# Agenda

1. Introduction
2. Problem description
3. Proposed solution
4. Simulated examples
5. Summary & conclusions

# Introduction 1/3

- Choice of a radar's **pulse repetition interval (PRI)** has great influence on target detection and tracking performance
- Interval might be constant:

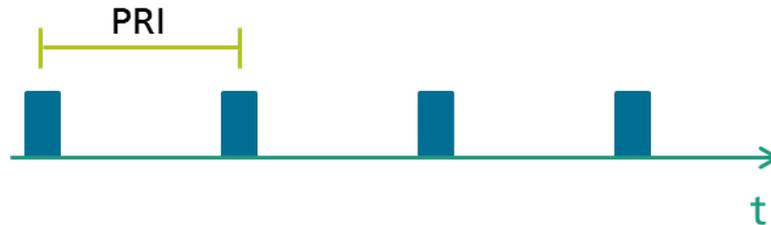


- Or with some modulation:

- (3-level stagger)

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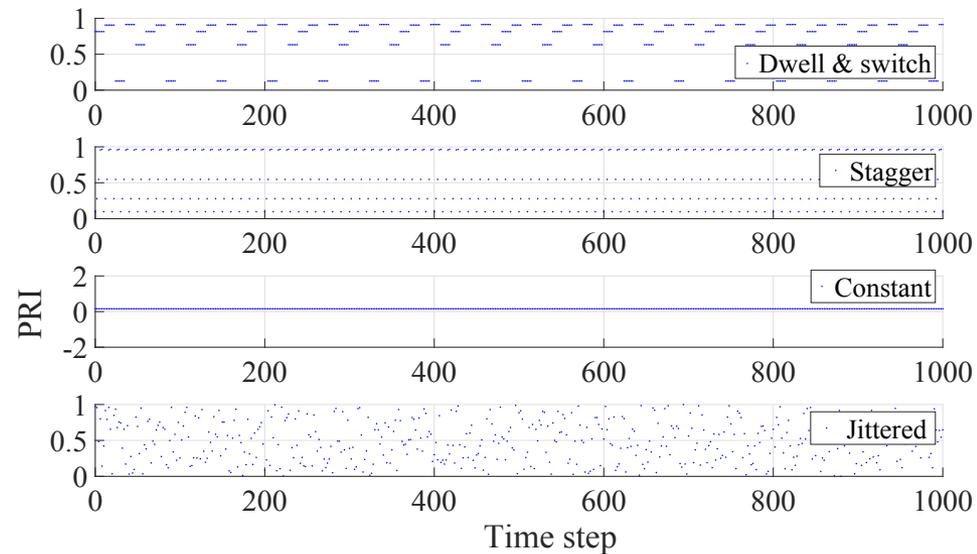
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# Introduction 2/3

- Classification of **pulse repetition interval modulation** important for electronic warfare systems:
  - Significant knowledge about the observed emitter
  - Improvement of own electronic warfare system functions
- Literature: Standard PRI modulation types only
  - Dwell & switch, stagger, constant, jittered, **complex**

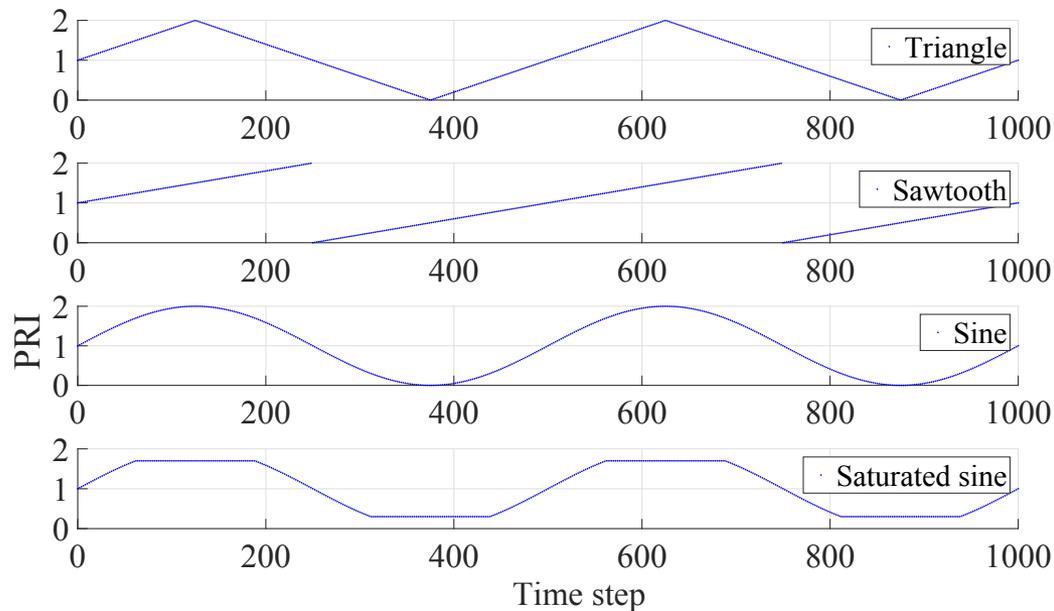
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# Introduction 3/3

- Automatic classification of **complex PRI modulation sub-types** remains unaddressed
  - Common: Triangle, sawtooth, sine, and saturated sine



# Problem description 1/2

Consider a scenario where:

- A receiver observes an area of interest and records pulses emitted from different radars
- The received pulses are deinterleaved, i.e sorted by emitter
  - Deinterleaving is a complex topic itself - not in scope
  - Effects accounted for by considering spurious and missing pulses

Problem formulation:

Does the received signal exhibit a complex PRI modulation?

If yes, of which sub-type: sawtooth, triangle, sine, or saturated sine?

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# Problem description 2/2

This is essentially a *multi-class classification* or *multiple hypotheses testing* problem:

- Hypothesis  $H_1$ : class  $C_1$ , i.e. sawtooth modulation
- Hypothesis  $H_2$ : class  $C_2$ , i.e. triangle modulation
- Hypothesis  $H_3$ : class  $C_3$ , i.e. sine modulation
- Hypothesis  $H_4$ : class  $C_4$ , i.e. saturated sine modulation
- Hypothesis  $H_0$ : class  $C_0$ , i.e. none of the above

We desire high **probability of correct classification**:

$$P_C^j = P(C^* = C_j | C_{\text{true}} = C_j), j = 1, \dots, 4$$

and low **probability of misclassification**:

1. Modulation type  $j$  is classified as some other type

$$P_{M-v1}^j = P(C^* \neq C_j | C_{\text{true}} = C_j), j = 1, \dots, 4$$

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# Proposed solution

**Input:** TOA difference of pulses  $\Delta t$ , cross-correlation threshold  $c_{min}$

**Output:** Complex modulation type hypothesis decision  $H_j : C^* = C_j, j \in \{0, 1, 2, 3, 4\}$

```
1:  
2:  
3:  
4:  
5:  
6:  
7:  
8: if then  
9:  
10: else  
11:  
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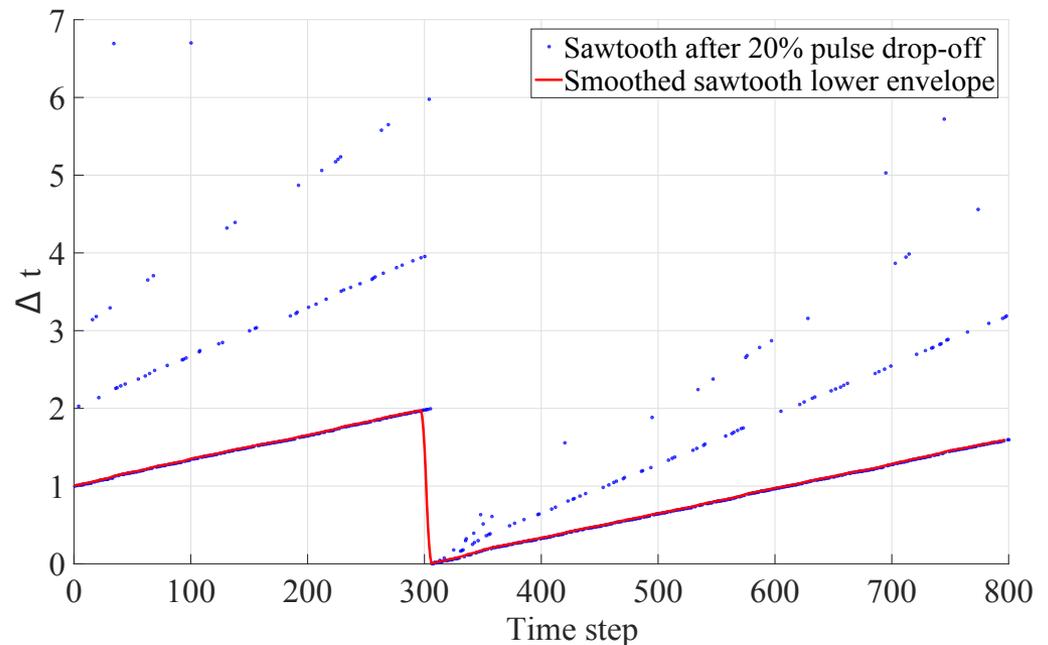
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## Elimination of the effect of lost pulses



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3: evaluate  $(\widehat{\Delta t} \star \widehat{\Delta t})$ , i.e. its autocorrelation

4: find the period of  $\Delta t$  using the peaks of  $(\widehat{\Delta t} \star \widehat{\Delta t})$

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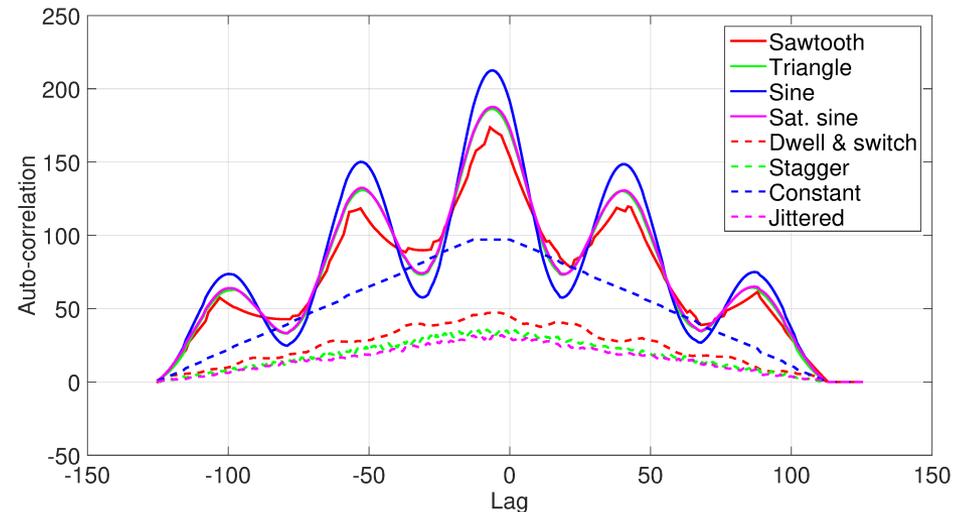
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Complex PRI modulation induces distinct peaks

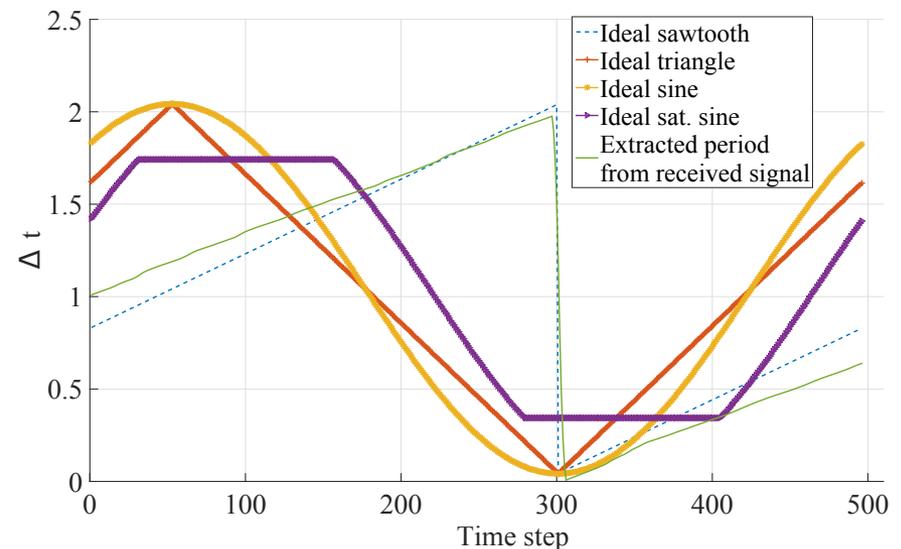


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- 4: find the period of  $\Delta t$  using the peaks of  $(\widehat{\Delta t} \star \widehat{\Delta t})$
- 5: extract a period from  $\widehat{\Delta t}$
- 6: create ideal signals  $\Delta t_j^*, j = 1, \dots, 4$
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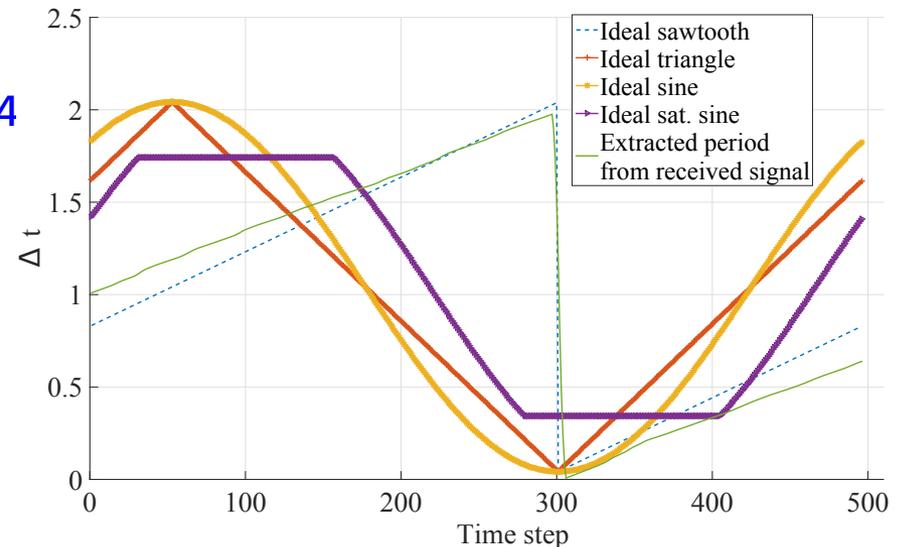


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- 6: create ideal signals  $\Delta t_j^*, j = 1, \dots, 4$
- 7: find  $j^* = \arg \max_j [(\widehat{\Delta t} \star \Delta t_j^*)], j = 1, \dots, 4$
- 8: if  $(\widehat{\Delta t} \star \Delta t_{j^*}^*) > c_{min}$  then
- 9: choose hypothesis  $H_{j^*} : C^* = C_{j^*}$
- 10: else
- 11: choose hypothesis  $H_0 : C^* = C_0$
- 12: end if



# Simple example 1: Favourable case

In this case we assume **very reliable** prior information:

- Normalized cross-correlation threshold  $c_{min} = 0.8$
- Duration of the emitted signal  $D = 200$  time units
- We sample 1.8 periods of the signal
- Drop-out ratio of 10%, i.e. 10% of the emitted pulses are lost
- Saturation of sat. sine is known to be 0.7

PRI mod.	$P_C$	$P_{M-v1}$	$P_{M-v2}$
Sawtooth	0.96	0.01	0.0001
Triangle	0.88	0.12	0.22
Sine	0.83	0.17	0.11
Sat. sine	0.84	0.16	0.09

PRI mod.	$P_C$	$P_{M-v1}$	$P_{M-v2}$
Dwell & switch	N/A	0.001	N/A
Stagger	N/A	0	N/A
Constant	N/A	0	N/A
Jittered	N/A	0.0003	N/A

# Simple example 2: Unfavourable case 1

In this case we assume reliable prior information but **more pulses are lost**:

- Normalized cross-correlation threshold  $c_{min} = 0.8$
- Duration of the emitted signal  $D = 200$  time units
- We sample 1.8 periods of the signal
- **Drop-out ratio of 20%**, i.e. 20% of the emitted pulses are lost
- Saturation of sat. sine is known to be 0.7

PRI mod.	$P_C$	$P_{M-v1}$	$P_{M-v2}$
Sawtooth	0.9	0.05	0.0001
Triangle	0.64	0.36	0.25
Sine	0.79	0.2	0.34
Sat. sine	0.6	0.4	0.22

PRI mod.	$P_C$	$P_{M-v1}$	$P_{M-v2}$
Dwell & switch	N/A	0.007	N/A
Stagger	N/A	0.0008	N/A
Constant	N/A	0	N/A
Jittered	N/A	0.003	N/A

## Simple example 3: Unfavourable case 2

In this case we assume **unreliable** prior information:

- Normalized cross-correlation threshold  $c_{min} = 0.8$
- Duration of the emitted signal  $D = 100$  time units
- We sample **1.5 periods** of the signal
- **Drop-out ratio of 20%**, i.e. 20% of the emitted pulses are lost
- Saturation of **ideal sat. sine** is **0.8** instead of the true value 0.7

PRI mod.	$P_C$	$P_{M-v1}$	$P_{M-v2}$
Sawtooth	0.9	0.07	0.03
Triangle	0.06	0.94	0.02
Sine	0.06	0.94	0.11
Sat. sine	0.9	0.09	0.95

PRI mod.	$P_C$	$P_{M-v1}$	$P_{M-v2}$
Dwell & switch	N/A	0.02	N/A
Stagger	N/A	0.002	N/A
Constant	N/A	0	N/A
Jittered	N/A	0.02	N/A

# In depth look into the performance

The following settings were used:

- Normalized cross-correlation threshold  $c_{min} = 0.8$
- Duration of the emitted signal  $D = 1000$  time units
- Mean **pulse repetition interval**  $PRI \in \{0.1, 0.25, 0.5, \dots, 3, 3.5, 4\}$  time units
  - Higher value means less pulses emitted in the same time
- **Number of observed signal periods**  $D/T \in \{1, 1.1, 1.2, \dots, 3\}$ 
  - Ratio of emitted signal duration  $D$  and signal period  $T$
  - Higher value means less pulses per period in the same observation time

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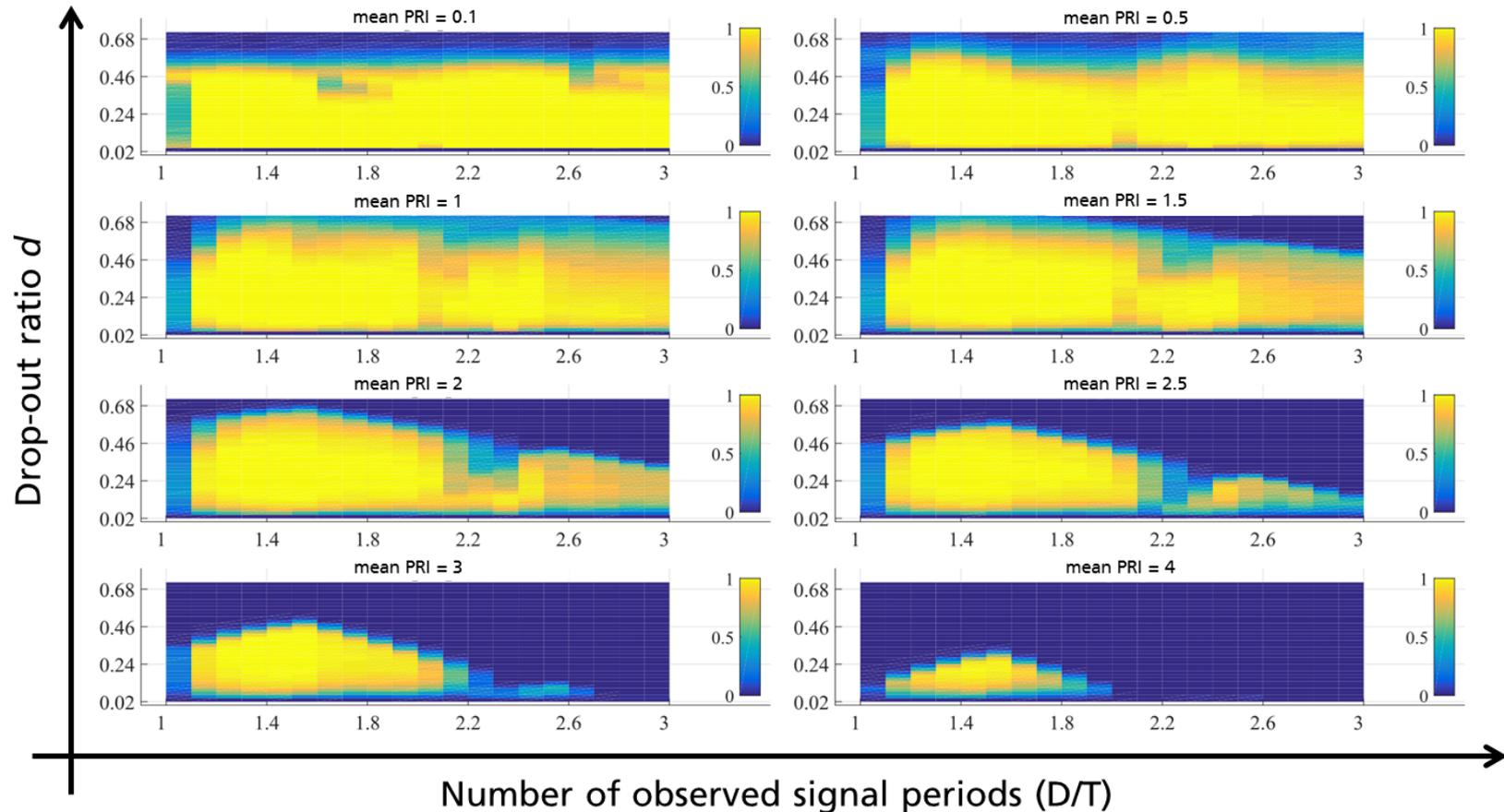
- **Drop-out ratio**  $d \in \{0, 0.02, 0.2, \dots, 0.7\}$
- 1000 Monte Carlo runs
- Pulses randomly dropped at each run based on the drop-out ratio

We examine the:

- Probability of correct classification
- Both definitions of the probability of misclassification

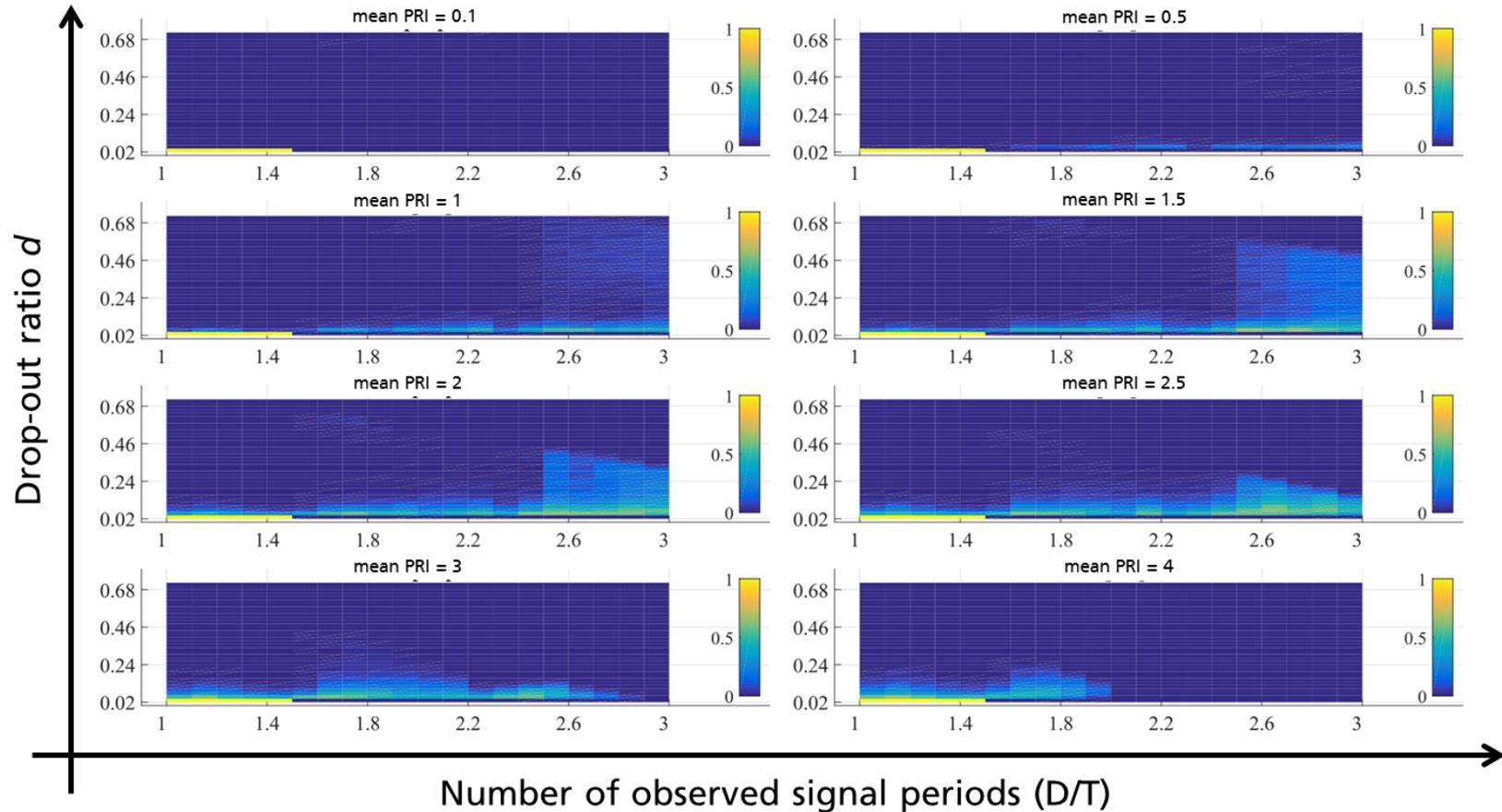
# Example: Sawtooth PRI modulation

Very high **probability of correct classification**  $P_C$  over a broad range of signal reception settings.



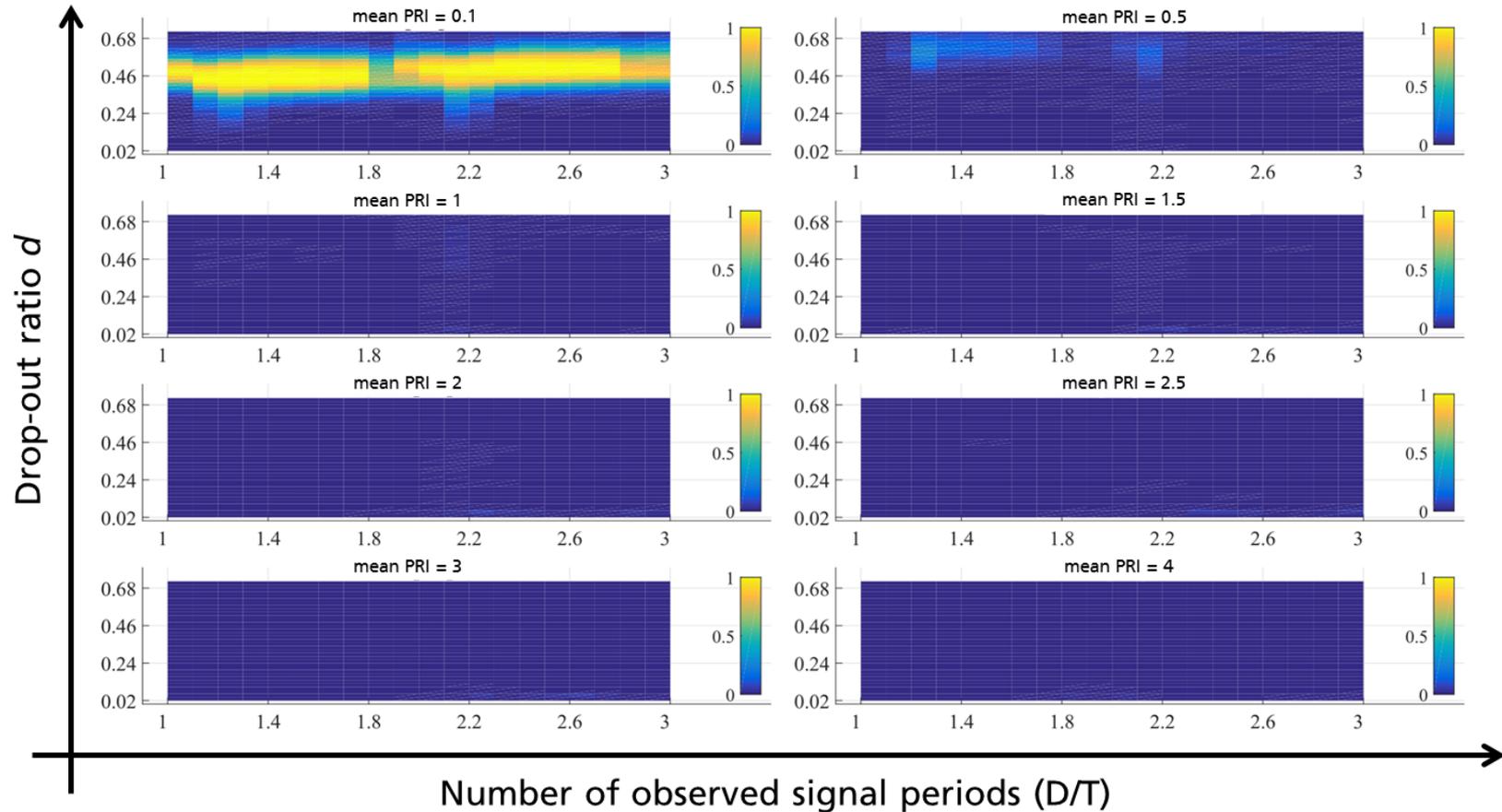
# Example: Sawtooth PRI modulation

Very low probability that sawtooth is classified as another complex modulation type  $P_{M-v1}$  over a broad range of signal reception settings.

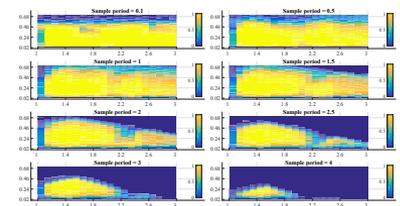


# Example: Sawtooth PRI modulation

Very low probability that other modulation types are classified as sawtooth  $P_{M-v2}$  over a broad range of signal reception settings.

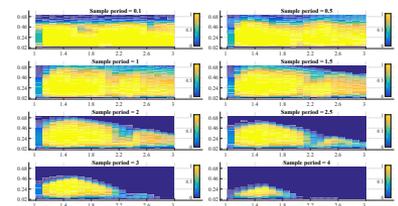


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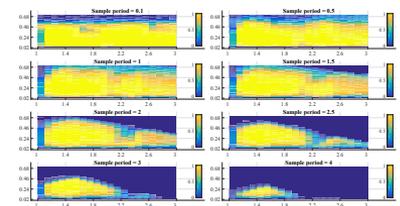
- Sawtooth modulation correctly classified in almost all cases
- Non-complex PRI modulations practically never identified as complex
- Most false classifications of triangle and sine are due to confusion with saturated sine
- Crucial part: Reliable extraction of the lower envelope of the received signal
  - Lower envelope should resemble one of the ideal complex modulation types
  - At least 1.2 periods should be observed
- Best performance for:  $D/T \in \{1.2, \dots, 2\}$ ,  $meanPRI \in \{0.1, \dots, 2\}$ ,  $d \in \{0.02, \dots, 0.46\}$
- Prior information about the received signal crucial for its correct classification
  - Knowledge about the signal period can be used for adapting the observation duration
- Significant pulse drop-out ratios can be tolerated
  - Up to 50% under some favourable conditions

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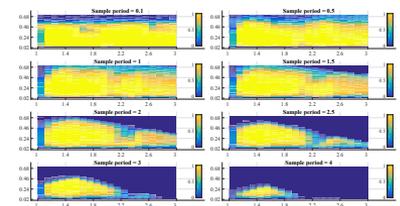
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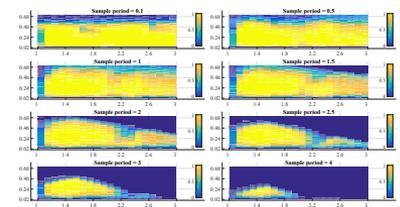
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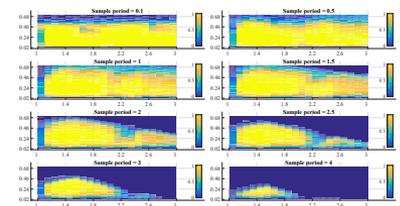
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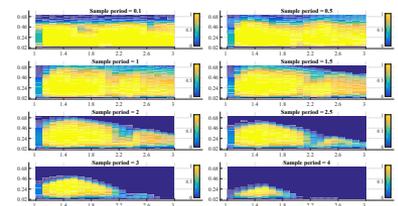
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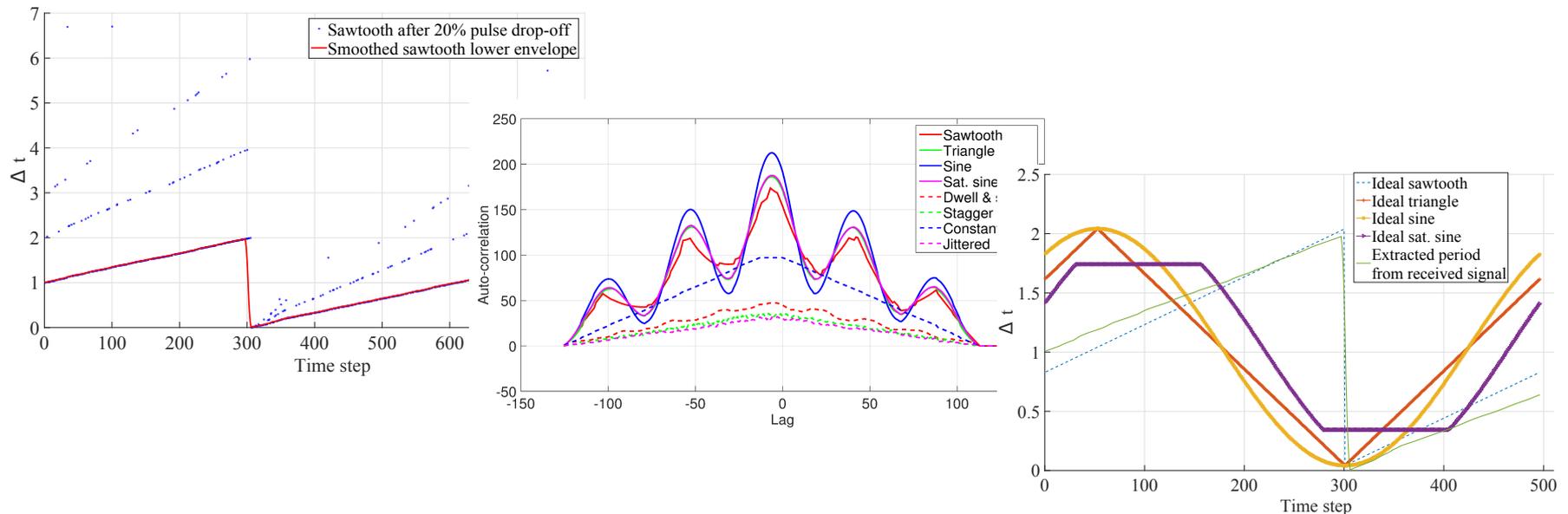
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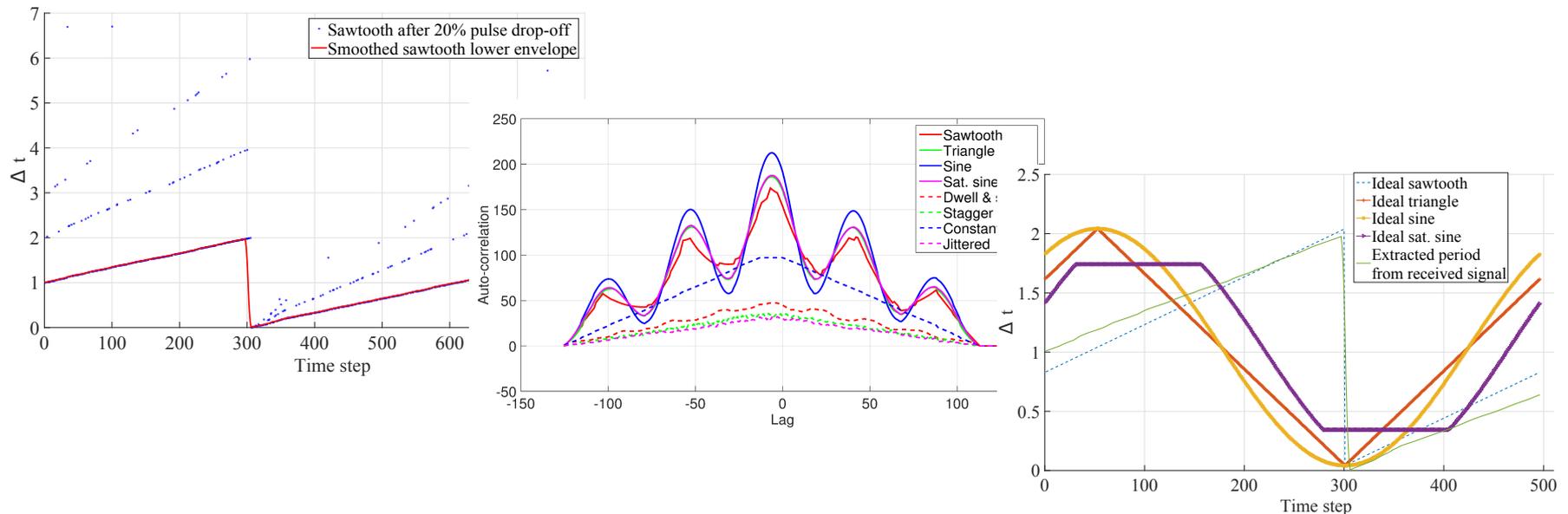
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- First algorithm in the open literature that classifies complex PRI modulation types
- Classification of complex PRI modulation with good statistics under varying signal reception conditions
- Information from an emitter database plays a crucial role
- Almost complete rejection of signals having non-complex PRI modulation
- Low computational complexity algorithm



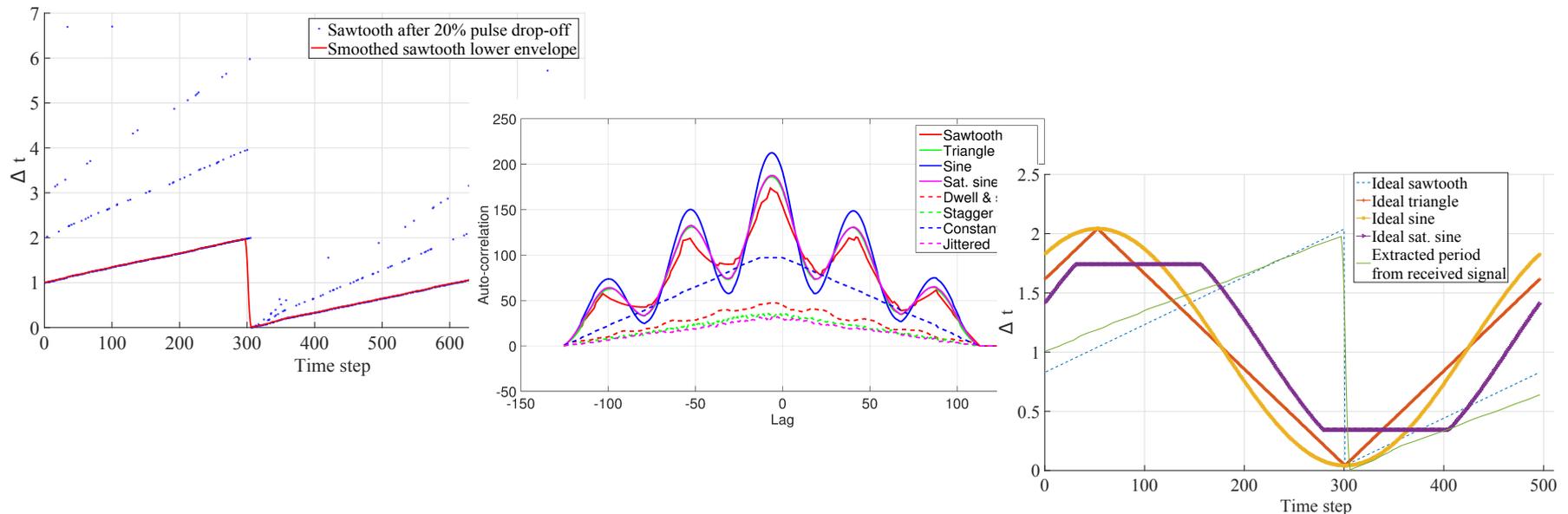
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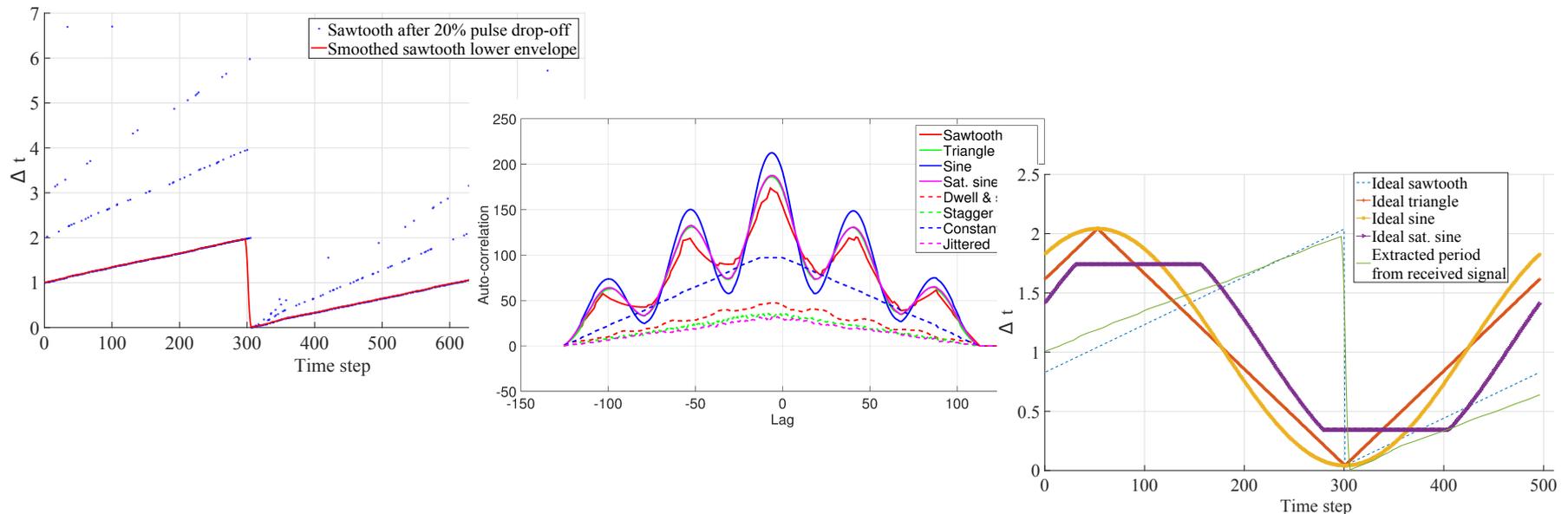
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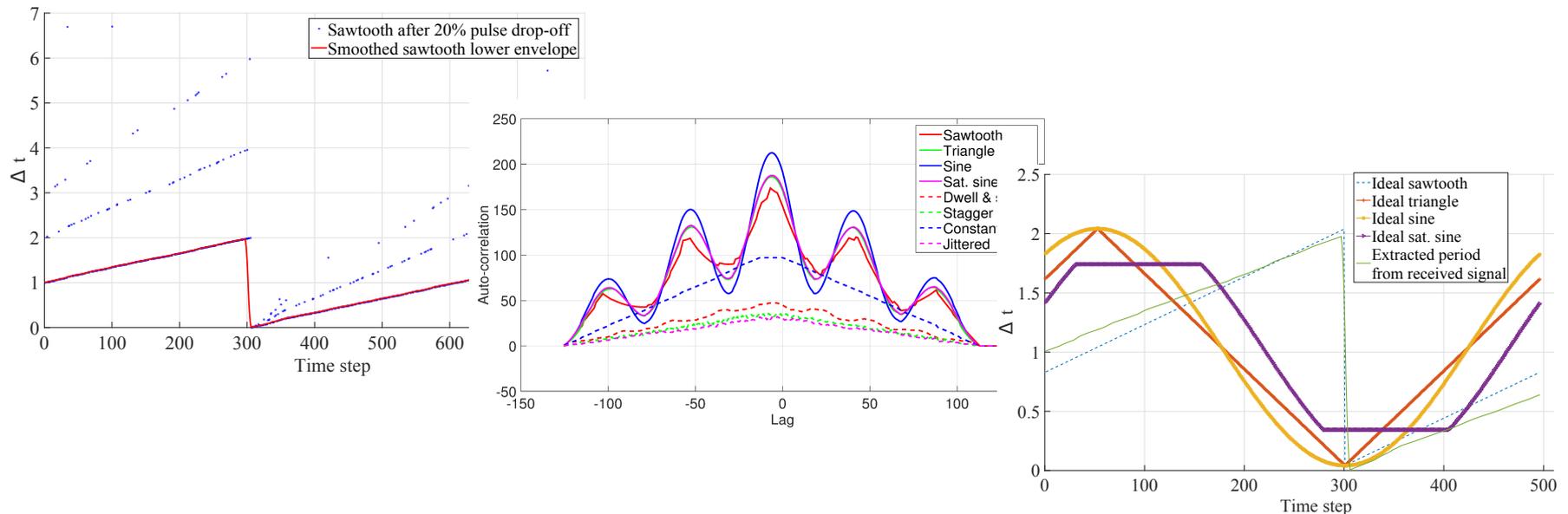
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Thank you  
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Fotios Katsilieris, Sabine Apfeld, Alexander Charlish

e-mail: *Fotios.Katsilieris@airbus.com*

*{Sabine.Apfeld, Alexander.Charlish} @fkie.fraunhofer.de*