# State Estimation of the Spread of COVID-19 in Saudi Arabia using Extended Kalman Filter


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## Highlight

- We consider the nonlinear epidemiological model in the Susceptible-Exposed-Infected-Quarantined-Recovered-Deceased (SEIQRD).
- Based on the recursive estimator extended Kalman filter (EKF) we predict the evolution of the COVID-19 pandemic over long term behaviour in Saudi Arabia using SEIQRD model.
- We adopt the nested sampling algorithm for parameter estimation and uncertainty quantification.
- Simulation results show that the EKF predicts the evolution of the directly measured variables i.e. the total death (D) and active case (I) and unmeasurable states such as exposed cases, quarantine cases, and recovered cases.

## The SEIQRD model

$$
\begin{aligned}
\frac{dS}{dt} &= -\beta IS + \alpha R \\
\frac{dE}{dt} &= \beta IS - \epsilon E \\
\frac{dI}{dt} &= \epsilon E - \gamma I - qI - dI \\
\frac{dQ}{dt} &= qI - q_t Q - dQ \\
\frac{dR}{dt} &= \gamma I + q_t Q - \alpha R \\
\frac{dD}{dt} &= dI + dQ
\end{aligned}
\tag{1}
$$

## Methods

- The Extended Kalman filter (EKF) algorithm is a recursive estimator of the nonlinear systems.
- The EKF consists of two stages: prediction and update. Predicting the state estimates and the error covariance then updating these estimates after the measurement is utilized.
- Summarise the algorithm as:

$$\hat{x}_t = \hat{x}_{t-1} + K_t \left( y_t - h_t \left( \hat{x}_t \right) \right) \tag{2}$$

$$K_t = P_t^- H_t^T \left( H_t P_t^- H_t^T + \Omega_t \right)^{-1} \tag{3}$$

$$P_t = \left( I - K_t H_t \right) P_{t-1} \tag{4}$$

*for more details see [1]*

## Parameter Estimation and Simulation of COVID-19 Spread in Saudi Arabia

We analysed the Saudi Arabia COVID-19 data with active cases (I) and death cases (D) between 15 February 2020 to 17 March 2022 based on the openly available dataset in [2]. We used the nested sampling algorithm [3] with Markov Chain Monte Carlo (MCMC) random walk for the live points to draw samples from the likelihood surface to estimate model parameters from the posterior distribution.

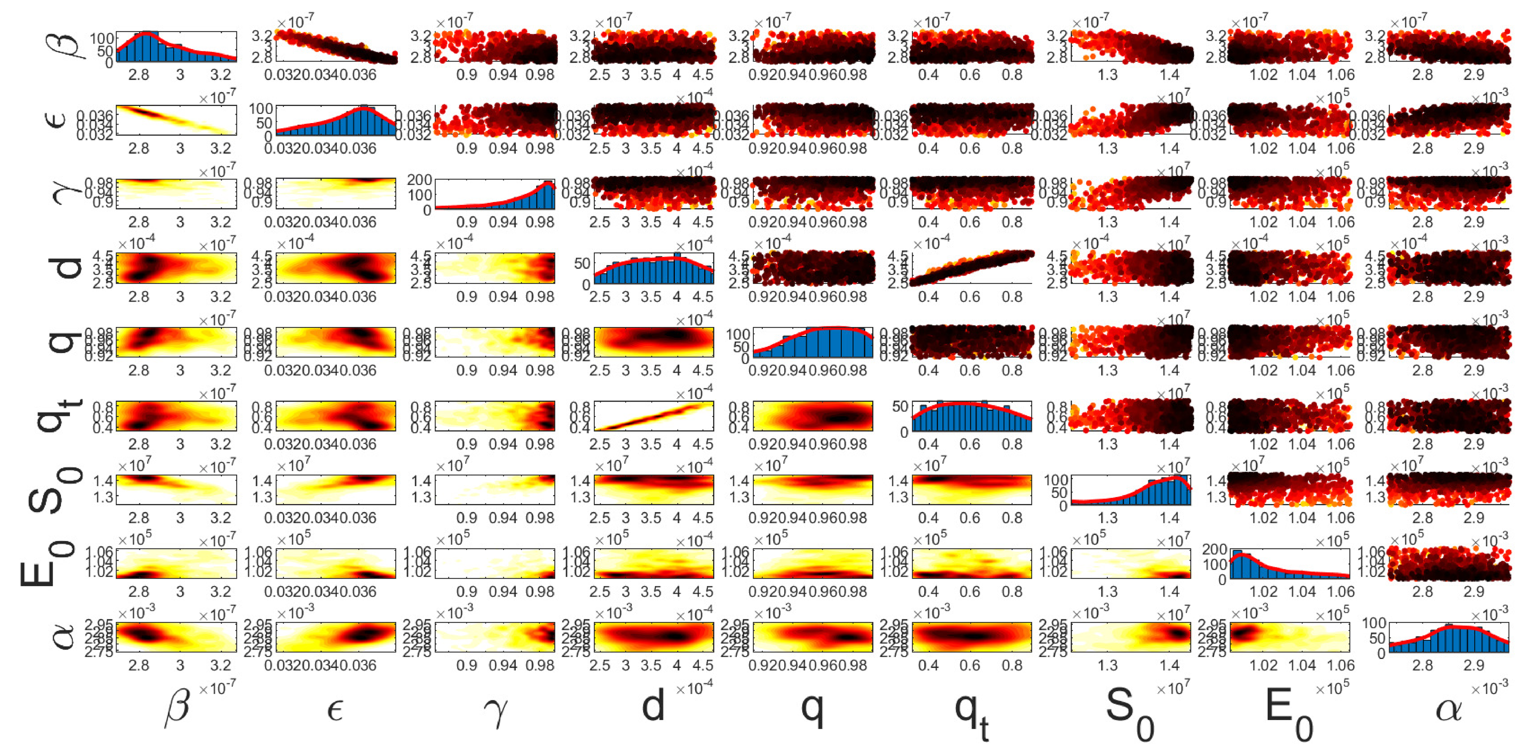

Figure 1:Posterior distribution of the proposed SEIQRD model parameters with the reinfection term.

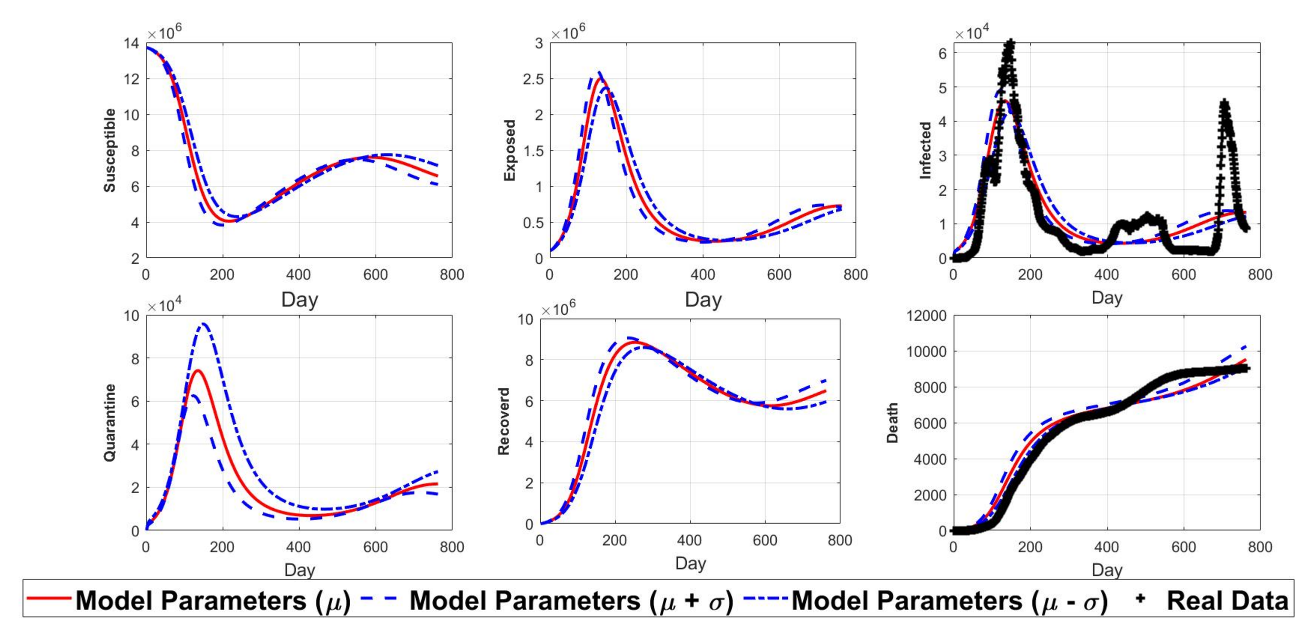


Figure 2:The estimated uncertainty bounds of the posterior distribution of the model parameters.

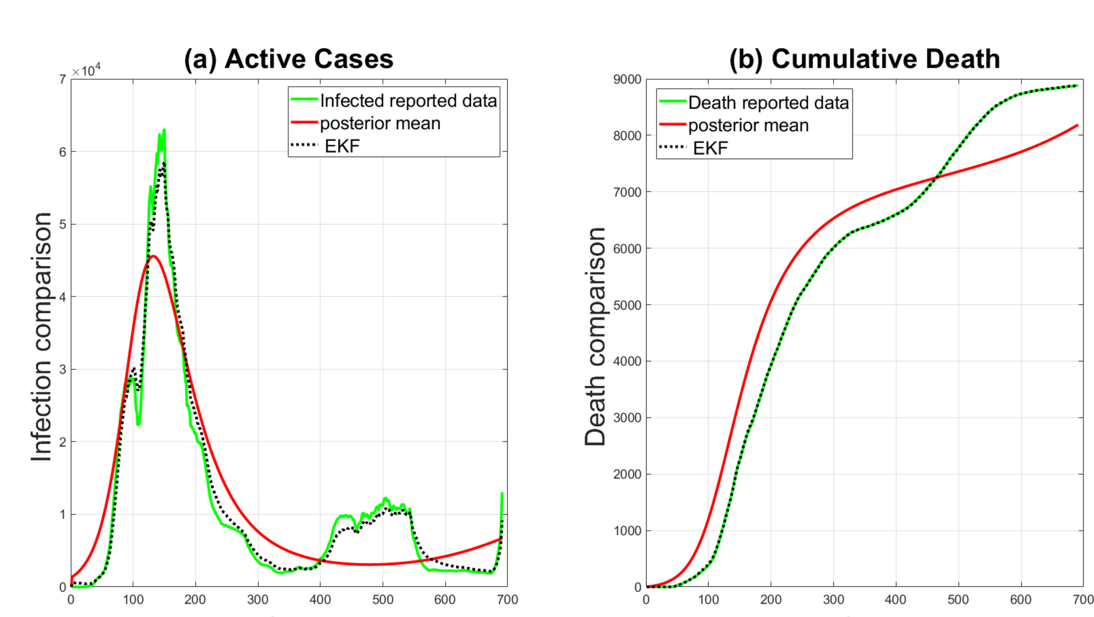


Figure 3:Comparison of the state estimation based on EKF with real data in Saudi Arabia: (a) active cases (b) cumulative death.

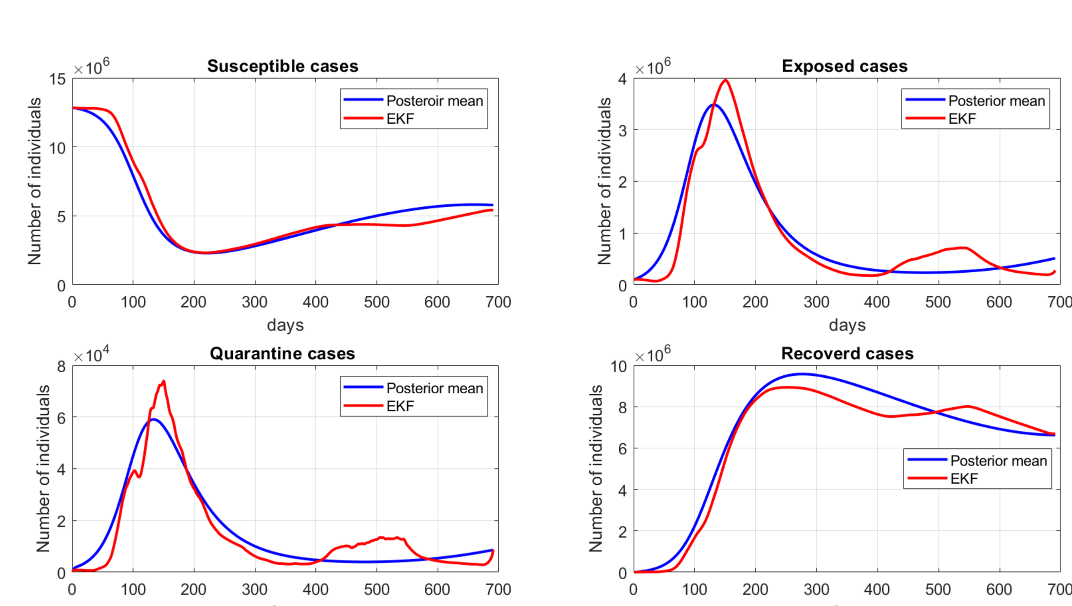


Figure 4:Comparing estimated variables of the Susceptible, Exposed, Quarantined and Recovered cases in Saudi Arabia.

Table 1:Root Mean Square Error of the EKF-based on SEIQRD Model

| Covariance Matrices | Infected Error | Death Error |
|---|---|---|
| $\Xi = 1$ , $\Omega =$ diag[10, 10] | 196.442 | 0.74698 |
| $\Xi = 0.01$ , $\Omega =$ diag[10 10000] | 212.3171 | 42.2808 |
| $\Xi = 500$ , $\Omega =$ diag[100, 1000] | 56.3065 | 0.27098 |

Table 2:Mean Posterior of the proposed SEIQRD Model Parameters for the Saudi Arabia COVID-19 Data

| parameter | Value | Description |
|---|---|---|
| $\beta$ | 2.92 $\times 10^{-7}$ | infection rate |
| $\alpha$ | 0.0028 | reinfection rate |
| $\epsilon$ | 0.0353 | 1/incubation period |
| $q$ | 0.9593 | quarantine rate |
| $qt$ | 0.5939 | time period of quarantine |
| $d$ | 3.5853$\times 10^{-4}$ | death rate |
| $\gamma$ | 0.9586 | recovery rate |

## Conclusion

- We present a new epidemiological model of the SEIQRD form with reinfection to understand the impact of COVID-19 based on active and death cases data in Saudi Arabia.
- Nested sampling algorithm based posterior mean parameters were used in the SEIQRD model for dynamic simulations.
- EKF was applied to estimate the dynamics of COVID-19 and simulation results show that the EKF can predicts the evolution the unmeasurable state variables and the EKF in the long term prediction is more accurate than the fitted model.

## References


[1] Dan Simon.
*Optimal state estimation: Kalman, H infinity, and nonlinear approaches.*
John Wiley & Sons, 2006.

[2] Worldometer.
Covid-19 worldometer daily snapshots, 2022.

[3] John Skilling.
Nested sampling for general bayesian computation.
*Bayesian Analysis*, 1(4):833–859, 2006.


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