Enhanced Space-Time Covariance Estimation Based on System Identification Approach

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Problem Statement

For many broadband array processing problems, estimate of space-time covariance matrix (STCM) is required. It is usually estimated via an un-biased estimator from sensor's data directly. However under some conditions, we may have control over the sources to permit system identification for a better STCM estimate. Hence, we present simulation results that quantify the accuracy of both estimates for comparison.

Source Model and STCM

Fig.1 shows broadband signal received at an array of M sensors in form of vector $\mathbf{x}[n] \in \mathbb{C}^M$ from L uncorrelated sources $u_l[n], l = 1, \dots, L$ convolutively mixed via channel matrix $\mathbf{H}[n] \in \mathbb{C}^{M\times L}$ which can be given as

$$\mathbf{H}[n] = \begin{bmatrix} h_{11}[n] & h_{12}[n] & \dots & h_{1L}[n] \\ h_{21}[n] & h_{22}[n] & \dots & h_{2L}[n] \\ \vdots & \vdots & \ddots & \vdots \\ h_{M1}[n] & h_{M2}[n] & \dots & h_{ML}[n] \end{bmatrix}$$

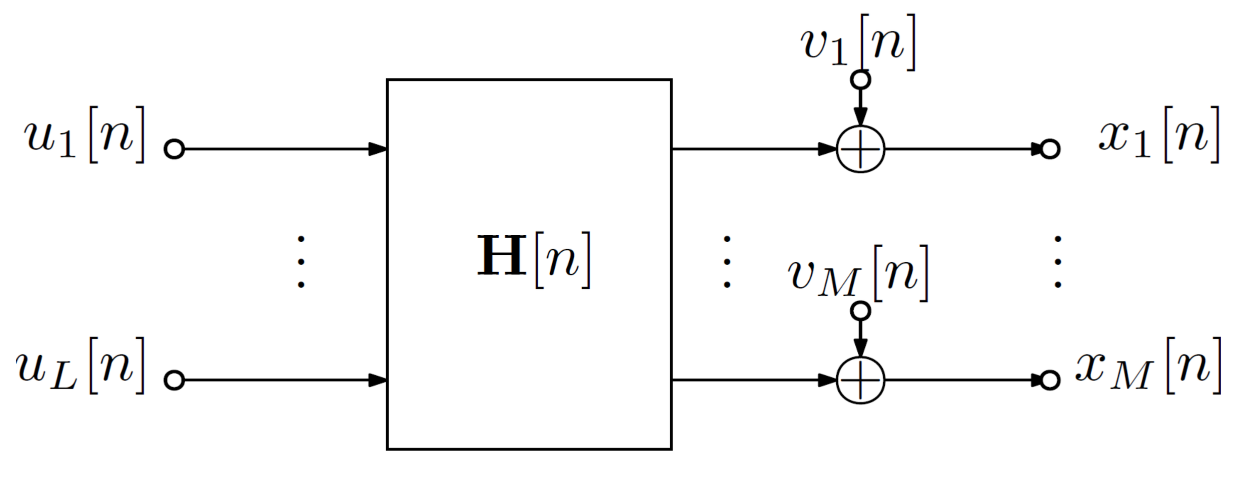

Fig. 1. Source model for STCM estimation

Assuming

$$\mathcal{E}\{\boldsymbol{u}[n]\boldsymbol{u}^{\mathrm{H}}[n-\tau]\} = \boldsymbol{I}_M\delta[\tau]$$

$$\mathcal{E}\{\boldsymbol{v}[n]\boldsymbol{v}^{\mathrm{H}}[n-\tau]\} = \sigma_v^2\boldsymbol{I}_M\delta[\tau]$$

where $\mathcal{E}\{\cdot\}$ represents the expectation operator, STCM can either be represented as an expectation of sensors data vector $\mathbf{x}[n] \in \mathbb{C}^M$ as:

$$\mathbf{R}[\tau] = \mathcal{E}\{\mathbf{x}[n]\mathbf{x}^{\mathrm{H}}[n-\tau]\}$$

or tied to stable and causal matrix $\mathbf{H}[n]$ as

$$\mathbf{R}[\tau] = \sum_n \mathbf{H}[n]\mathbf{H}^{\mathrm{H}}[n-\tau] + \sigma_v^2\mathbf{I}_M\delta[\tau]$$

Each element of $\mathbf{R}[\tau] \in \mathbb{C}^{M\times M}$ is a cross-correlation sequence i.e.

$$r_{lm}[\tau] = \mathcal{E}\{x_l[n]x_m^*[n-\tau]\} \qquad (1)$$

$$r_{lm}[\tau] = \sum_n \sum_{k=1}^{L} h_{l,k}[n]h_{m,k}^*[n-\tau] + \sigma_v^2\delta[\tau]\delta[l-m] \qquad (2)$$

For (1), we use unbiased estimator and for (2), the system

Unbiased Estimator

If N snapshots of data are available i.e. $\mathbf{x}[n], n = 0, \dots, N-1,$ the un-biased estimator for (1) can be given as [1]

$$\hat{r}_{l,m}[\tau] = \begin{cases} \frac{1}{N-|\tau|}\sum_{n=0}^{N-|\tau|-1} x_l[n-\tau]x_m^*[n], \tau \geq 0 \\ \frac{1}{N-|\tau|}\sum_{n=0}^{N-|\tau|-1} x_l[n]x_m^*[n-\tau], \tau < 0 \end{cases}$$

Unbiased estimator treats the measurement noise as part of data and so its variance is independent of the SNR. For finite data, above assumption of un-correlated sources does not hold.

System Identification

If we have control over the source signals and known $u_l[n], l = 1, \dots, L$, channel matrix $\mathbf{H}[n]$ can be identified using SI via adaptive filter theory. Using Wiener solution [2], we identify M separate L-channel adaptive filters of length L_f as

$$\widehat{\boldsymbol{w}}_{m,opt} = \widehat{\mathbf{R}}^{-1}\widehat{\mathbf{p}}_m, \qquad m = 1, \dots, M$$

where $\widehat{\boldsymbol{w}}_{m,opt} = \begin{bmatrix} \widehat{\boldsymbol{h}}_{m1} \\ \vdots \\ \widehat{\boldsymbol{h}}_{mL} \end{bmatrix}$. The input sample covariance matrix $\widehat{\mathbf{R}}$ and vector $\widehat{\mathbf{p}}_m$ estimates $\mathcal{E}\{\boldsymbol{y}[n]\boldsymbol{y}^{\mathrm{H}}[n]\}$ and $\mathcal{E}\{\boldsymbol{y}[n]x_m[n]\}$ respectively over N-snapshots of data with $\boldsymbol{y}[n] = \begin{bmatrix} \boldsymbol{u}_1[n] \\ \vdots \\ \boldsymbol{u}_L[n] \end{bmatrix}$ and $\boldsymbol{u}_l[n] = \left[u_l[n], \dots, u_l[n-L_f+1]\right]^{\mathrm{T}}$. After SI, $\widehat{\mathbf{R}}[\tau]$ can be estimated via (2).

Estimate of $\sigma_{v,m}^2$ is obtained via minimum mean squared error

$$\hat{\sigma}^2{}_{v,m} = \hat{\sigma}_{x_m}^2 - \widehat{\mathbf{p}}_m^{\mathrm{H}}\mathbf{R}^{-1}\widehat{\mathbf{p}}_m$$

where $\hat{\sigma}_{x_m}^2$ denotes $x_m[n]$ power estimated over N-snapshots.

Results

We compare results for unbiased estimator with SI via metric

$$\zeta_n = \frac{\sum_\tau \left|\left|\mathbf{R}[\tau] - \widehat{\mathbf{R}}[\tau]\right|\right|_{\mathrm{F}}^2}{\sum_\tau \left|\left|\mathbf{R}[\tau]\right|\right|_{\mathrm{F}}^2}$$

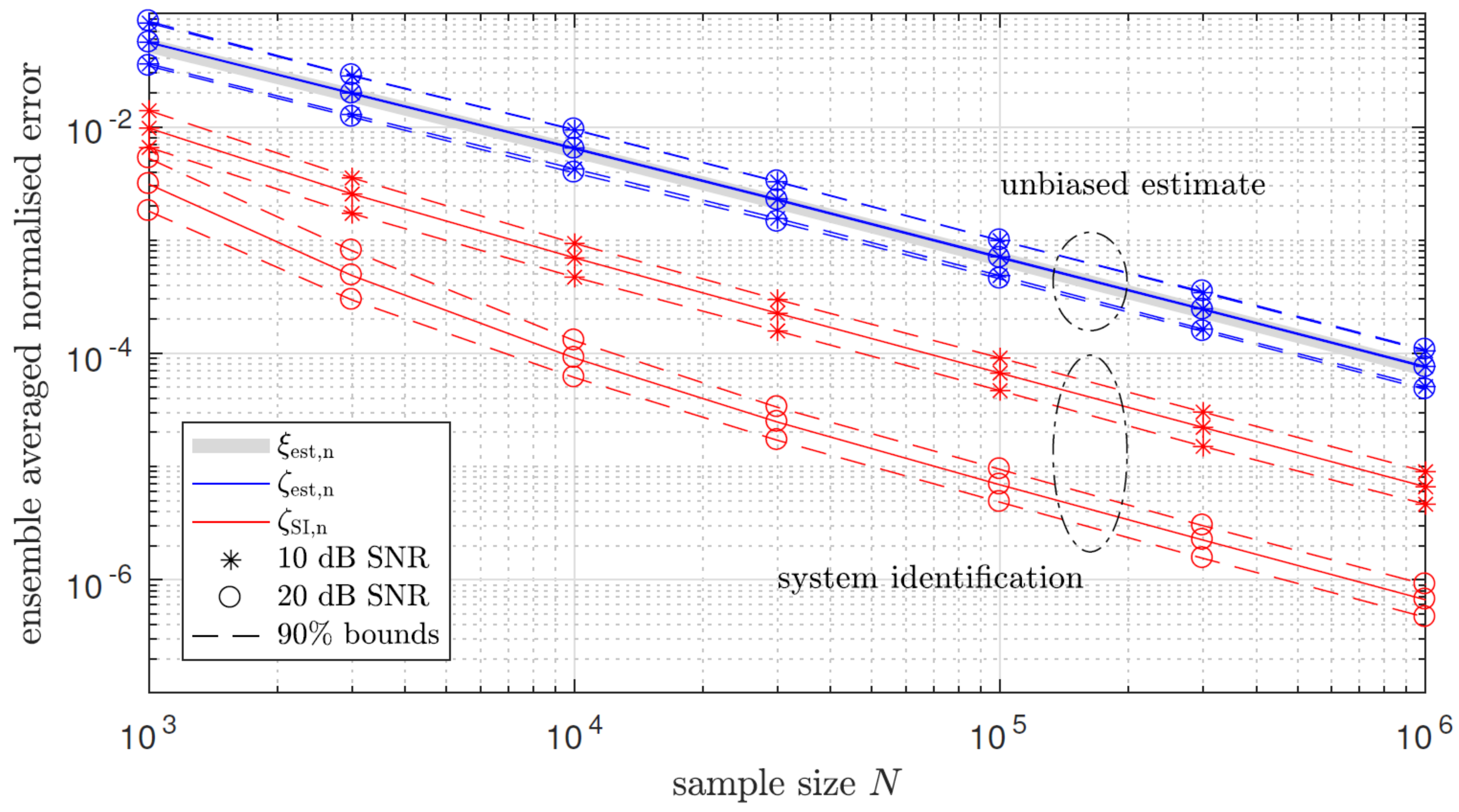

Fig.2. Estimation comparison for ensemble of $\boldsymbol{R}[\tau] \in \mathbb{C}^{2\times 2}$, showing the theoretical and measured error via unbiased estimator, $\xi_{est,n}$ and $\zeta_{est,n}$, respectively, as well as the measured error for SI, $\zeta_{SI,n}$

- SI performs significantly better than unbiased estimator at reasonable to high SNR (see Fig. 2) because with finite data at high SNR, channel matrix can be accurately identified.
- ξ_{SI} increases with decrease in SNR and eventually SI performance drops below the unbiased estimator at low SNR

Research Impact

Accurate STCM leads to

- lower perturbation of subspaces [3]
- accurate subspaces provide benefits for subspace-based techniques, such as broadband MUSIC for angle of arrival estimation [4]

[1]. C. Delaosa et al., 2019, ICASSP, 10.1109/ICASSP.2019.8683339
[2]. S. Haykin, Prentice Hall, Adaptive Filter Theory, 4th edition, 2002
[3]. C. Delaosa et al., 2020, SSPD, 10.1109/SSPD47486.2020.9272125
[4]. C. Delaosa et al., 2018, SAM, 10.1109/SAM.2018.8448482

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