High-Resolution DOA Estimation for Contiguous Target with Large Power Difference

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SSPD Conference

1. Contribution

Ability to estimate Direction-of-Arrival (DoA) for contiguous targets with

- Few sensors and snapshots
- Large power difference between the targets

This is achieved by re-weighting the ℓ_1-SVD with

- Capon-MUSIC, and
- Capon-MUSIC Group delay

Thus able to achieve high spatial resolution at high values of Signal-to-Interference-Noise-Ratio (SINR).

2. Introduction

Some common techniques for DOA estimation :

- Delay-Sum beamformer
- Capon beamformer
- MUSIC, ESPRIT and Min. Norm

The performance degrades significantly with :

- Less number of sensors and snapshots
- Large power differences between the contiguous targets.

In practical scenarios like :

- Passive SONAR, echos from the targets is several dB's lower than ships self-noise
- When one target is near and the other far away but contiguous in angular space.

3. Earlier Techniques

Some earlier attempts to estimate DoA's for targets with large power differences :

- Jamming method
- Constrained MUSIC and Capon-MUSIC
- Eigen-beam mCapon
- Robust sparse asymptotic minimum variance (RSAMV) algorithm
- Robust Orthogonal Projection

4. Signal Model

For J targets impinging upon N sensor Uniform Linear Array from $\theta_1 \ldots, \theta_J$ directions. The data received as:

$$\mathbf{y}(t) = \sum_{j=1}^{J} \mathbf{a}(\theta_j) s_j(t) + \mathbf{n}(t);\ t = 1, \ldots, L \quad (1)$$

$\mathbf{a}(\theta_j) \rightarrow$ array steering vector; $s_j(t) \rightarrow$ amplitude of j^{th} target; $\mathbf{n}(t) \rightarrow$ Complex Gaussian noise

In matrix form Eq.1 is written as :

$$\mathbf{Y} = \mathbf{AS} + \mathbf{N} \quad \in \mathbb{C}^{N \times L}$$

Covariance matrix of $\mathbf{y}(t)$ for L snapshots is given as:

$$\hat{\mathbf{R}} = \frac{1}{L} \sum_{t=1}^{L} \mathbf{y}(t)\mathbf{y}(t)^H = \frac{1}{L}\mathbf{Y}\mathbf{Y}^H$$

5. Proposed Method

With over-complete set of steering vectors$\rightarrow$ $\mathbf{A}_\Omega = \{\mathbf{a}(\theta_{\Omega,k}) \,|\, k \in [1, K]\} \in \mathbb{C}^{N\times K}$, for K DoA bases$\rightarrow$ $\theta_\Omega = \{\theta_{\Omega,k} \,|\, \forall k\}$ includes the J target DoAs θ [$J \ll K$].

- Computing the SVD as $\mathbf{Y} = \mathbf{U}\Lambda\mathbf{V}$, and Group-delay as:

$$\nabla[\angle\{\mathbf{A}_\Omega^H \mathbf{U}_N\}] = [\tau_A \ \tau_B]$$

$\mathbf{A}_\Omega = [\mathbf{A} \ \ \mathbf{B}]$ where $\mathbf{A} \in \mathbb{C}^{N\times J} \rightarrow$ true DoAs and $\mathbf{B} \in \mathbb{C}^{N\times(K-J)} \rightarrow$ other directions.

- The Group-delay weights are computed as

$$\mathbf{w}_{Group-delay})^{-1} = [[\tau_A^{(\ell_2)}]^T \ [\tau_B^{(\ell_2)}]^T]^T$$

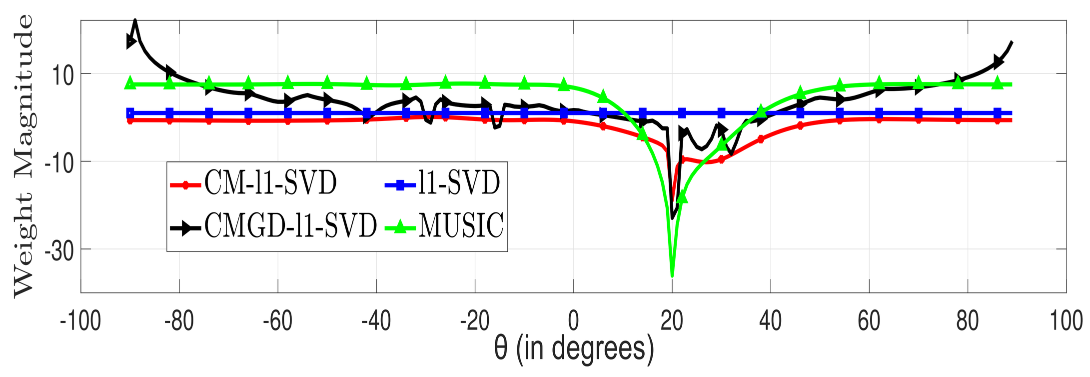

Figure 1: Angular spectrum for different weights at N=6, SINR=20 dB and targets at $20°$& $26°$

- Instability due to Group-delay spectrum (CMGD-ℓ_1-SVD), seen in Fig.1.
- Compensated by Hadamard product of the Capon-MUSIC [2] spectrum and GD weights as:

$$\mathbf{w}_{CMGD} = \frac{1}{\mathbf{w}_{Group-delay} \odot \mathbf{P}_{C-music}}$$

where

$$\mathbf{P}_{\mathbf{C-music}}(\Theta) = \frac{\mathbf{a}_\theta{}^H \hat{\mathbf{R}}^{-1} \mathbf{a}_\theta}{\mathbf{a}_\theta{}^H \mathbf{U_n} \mathbf{U_n}{}^H \mathbf{a}_\theta}$$

- Diagonalizing weights $\rightarrow$ $\mathbf{W} = \text{diag}\,(\mathbf{w}_{CMGD})$ the minimization equation is solved as

$$\min \|\mathbf{W}\bar{\mathbf{s}}^{(\ell_2)}\|_1 \text{ subject to } \|\mathbf{YVE} - \mathbf{A}_\Omega \hat{\mathbf{S}}_\Omega \mathbf{VE}\|_F^2 \leq \eta^2$$

$\mathbf{W}\bar{\mathbf{s}}^{(\ell_2)} = \hat{\mathbf{s}}_\Omega \in \mathbb{R}^K \rightarrow$ row-wise ℓ_2-norm of $\hat{\mathbf{S}}_\Omega \in \mathbb{C}^{K\times L}$, indicating DoAs, $\mathbf{E} = [\mathbf{I}^J, \mathbf{0}]^{\mathrm{L}} \in \mathbb{C}^{L\times J}$ with $\mathbf{I}^J \rightarrow$ identity matrix and $\mathbf{0} \rightarrow (J \times (L-J))$ zero matrix, and $\eta \rightarrow$ regularization coefficient [1].

- Another attempt by re-weighting ℓ_1-SVD using Capon-MUSIC weights as:

$$\mathbf{w}_{\mathbf{C-music}} = \frac{1}{\mathbf{P}_{C-music}}$$

6. Performance Evaluation

Performance evaluations settings

- Multi-snapshot processing with $L = 200$
- Targets at $20°$ and $26°$ with step-size Δ=1°
- The number of sensors $N = 6$
- Evaluated over S= 10^3 Monte-Carlo simulations

6.1 Average Root Mean Square Error

- Capon-MUSIC Group-delay-ℓ_1-SVD (CMGD-ℓ_1-SVD) and Capon-MUSIC ℓ_1-SVD (CM-ℓ_1-SVD) achieve low ARMSE score than state-of-the-art methods with less sensors.
- Achieves lowest ARMSE score for less snapshots for contiguous targets ($20°$ and $26°$) with a power difference of 20 dB,
- In terms of SINR (strong target $\rightarrow$20 dB, weak target $\rightarrow$[20 to -5] dB), proposed methods achieve minimum ARMSE score.
- Target $T_1 \rightarrow 20°$ and $T_2 \rightarrow$ [$40°$ - $22°$] , proposed methods achieve lowest ARMSE score.

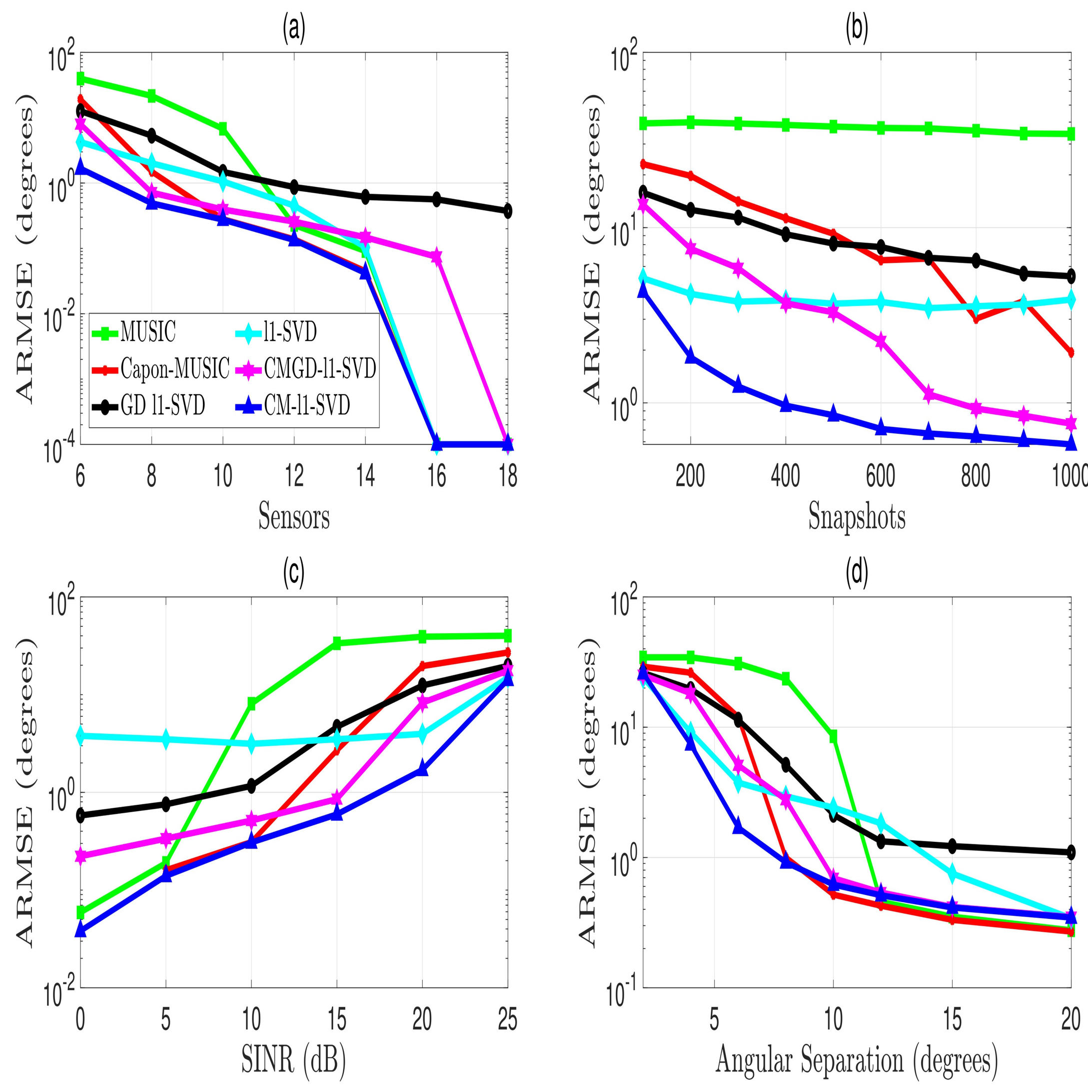

Figure 2: Comparison of proposed methods with state-of-the-art methods using (a) ARMSE Vs. No. of Sensors, (b) ARMSE Vs. Snapshots, (c) ARMSE Vs SINR and, (d) ARMSE Vs Angular Separation

6.2 Resolution Probability

- CM-ℓ_1-SVD outperforms all other algorithms in comparison.
- For angular separation $\rightarrow 2°$, resolution probability $\rightarrow$ 0.2 achieved.
- Observed both CM-ℓ_1-SVD and CMGD-ℓ_1-SVD achieve good resolution probability in SINR ranges.

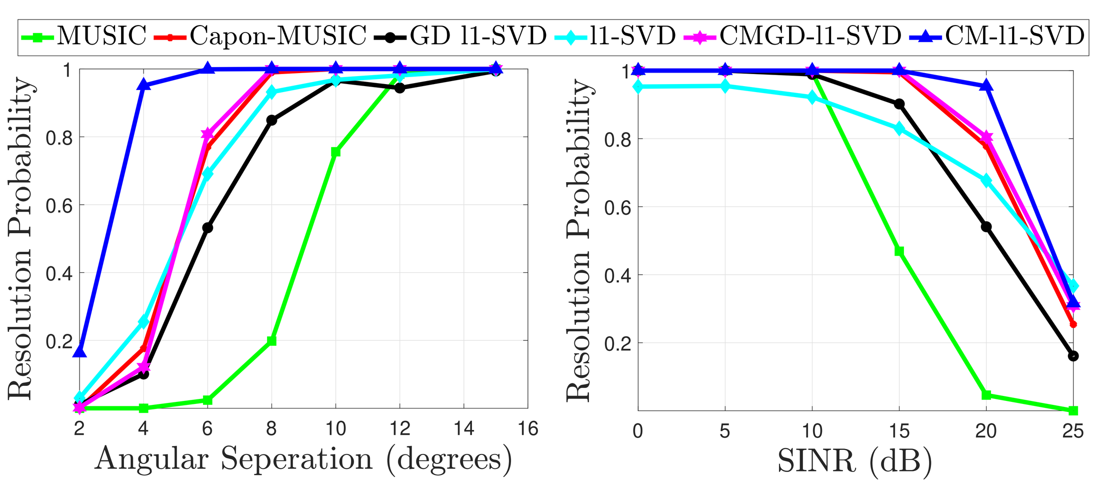

Figure 3: Plots for Resolution Probability evaluated at $N = 6, L = 200$.

References

[1] M. Ali, A. Koul, and K. Nathwani. Significance of group delay spectrum in re-weighted sparse recovery algorithms for doa estimation. *Digital Signal Processing*, 122:103388, 2022.

[2] Y. Gao, X. Jia, J. Xu, T.Long, and X Xia. A novel doa estimation method for closely spaced multiple sources with large power differences. In *2015 IEEE Radar Conference (RadarCon)*, pages 1276–1279, 2015.