An extension to the Frenet-Serret and Bishop Invariant Extended Kalman Filters for tracking accelerating targets

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Motivation

Tracking a manoeuvrable target requires kinematic models that are representative of target trajectories. The Frenet-Serret and Bishop formulae in equations (1) and (2) provide a concise means of characterising smooth curves and target trajectories using curvature and torsion parameters. The Frenet-Serret left-invariant extended Kalman filter (FS-LIEKF), first derived in [1], has been used to estimate the pose $\mathcal{X}_t \in SE(3)$ of a target by implementing an IEKF on $SE(3) \times \mathbb{R}^3$, using the Special Euclidean $SE(3)$ Lie group along with scalar parameters describing the shape and motion of the trajectory.

$$\begin{bmatrix} \dot{\mathbf{T}} \\ \dot{\mathbf{N}} \\ \dot{\mathbf{B}} \end{bmatrix} = u \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} \mathbf{T} \\ \mathbf{N} \\ \mathbf{B} \end{bmatrix} \quad (1) \qquad \begin{bmatrix} \dot{\mathbf{T}} \\ \dot{\mathbf{M}}_1 \\ \dot{\mathbf{M}}_2 \end{bmatrix} = u \begin{bmatrix} 0 & \kappa_1 & \kappa_2 \\ -\kappa_1 & 0 & 0 \\ -\kappa_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{T} \\ \mathbf{M}_1 \\ \mathbf{M}_2 \end{bmatrix} \quad (2)$$

An alternative formulation, the B-LIEKF, using a rotation-minimising Bishop frame has also been developed [2] with comparable performance to the original FS-LIEKF. However, both filters assume constant tangential velocity along with constant curvature and torsion and, although robust, do not have a component to account for tangential accelerations. This paper extends both previous filters to estimate the target's norm acceleration, by expanding the estimation space to $SE(3) \times \mathbb{R}^4$.

Methodology

The structure of the LIEKF algorithm on $SE(3) \times \mathbb{R}^4$ is shown below in Figure 1.

Algorithm 1 $SE(3) \times R^4$ LIEKF Algorithm with Spherical Measurements

Input: $\hat{\mathcal{X}}_{k-1|k-1}, \hat{\zeta}_{k-1|k-1}, P_{k-1|k-1}, Y_k, N_k^R, Q_k$

Output: $\hat{\mathcal{X}}_{k|k}, \hat{\zeta}_{k|k}, P_{k|k}$

$[\hat{\mathcal{X}}_{k|k-1}, \hat{\zeta}_{k|k-1}] = f(\hat{\mathcal{X}}_{k-1|k-1}, \hat{\zeta}_{k-1|k-1})$

$\Phi_k = \exp_m(A_t \Delta t)$

$P_{k|k-1} = \Phi_k P_{k-1|k-1} \Phi_k^T + Q_k$

if Y_k is available **then**

  $K_k = P_{k|k-1} H_k (H_k P_{k|k-1} H_k^T + N_k^R)^{-1}$

  $\hat{\mathcal{X}}_{k|k} = \hat{\mathcal{X}}_{k|k-1} \exp_{SE(3)}(K_k^{1:6}(Y_k - h(\hat{\mathcal{X}}_{k|k-1})))$

  $\hat{\zeta}_{k|k} = \hat{\zeta}_{k|k-1} + K_k^{7:10}(Y_k - h(\hat{\mathcal{X}}_{k|k-1}))$

  $P_{k|k} = (I_{10} - K_k H_k) P_{k|k-1}$

  return $\hat{\mathcal{X}}_{k|k}, \hat{\zeta}_{k|k}, P_{k|k}$

end if

Figure 1: $SE(3) \times \mathbb{R}^4$ LIEKF algorithm used for the FSa-LIEKF and Ba-LIEKF.

The state error propagation matrices (A_t) for the Ba-LIEKF and FSa-LIEKF are derived respectively as (3).

$$A_t = -\begin{bmatrix} 0 & -\hat{\kappa}_1 & -\hat{\kappa}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hat{\kappa}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \hat{\kappa}_2 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\hat{\kappa}_1 & -\hat{\kappa}_2 & 0 & 0 & -1 & 0 \\ 0 & 0 & -\hat{u}_t & \hat{\kappa}_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \hat{u}_t & 0 & \hat{\kappa}_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\hat{\kappa} & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ \hat{\kappa} & 0 & \hat{\tau} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \hat{\tau} & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\hat{\kappa} & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -\hat{u}_t & \hat{\kappa} & 0 & -\hat{\tau} & 0 & 0 & 0 & 0 \\ 0 & \hat{u}_t & 0 & 0 & \hat{\tau} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3)$$

The FSa-LIEKF and Ba-LIEKF are implemented alongside the original constant velocity variants and commonly used CA and CV EKFs.

Future Work

- Build the FS/B-LIEKF and FSa/Ba-LIEKFs into a multiple model algorithm such as the recently proposed Boxplus-IMM [3].
- Apply Invariant Unscented and Cubature Kalman filters to the tracking problem and implement on $\mathbb{D}S^3 \times \mathbb{R}^n$, the dual quaternion representation of $SE(3) \times \mathbb{R}^n$.
- Implement IEKFs into multi-target tracking filters such as GM-PHD and JPDA.

Simulation and Results

The sample trajectory in Figure 2 features helices and spiral segments difficult to track using a single model filter.

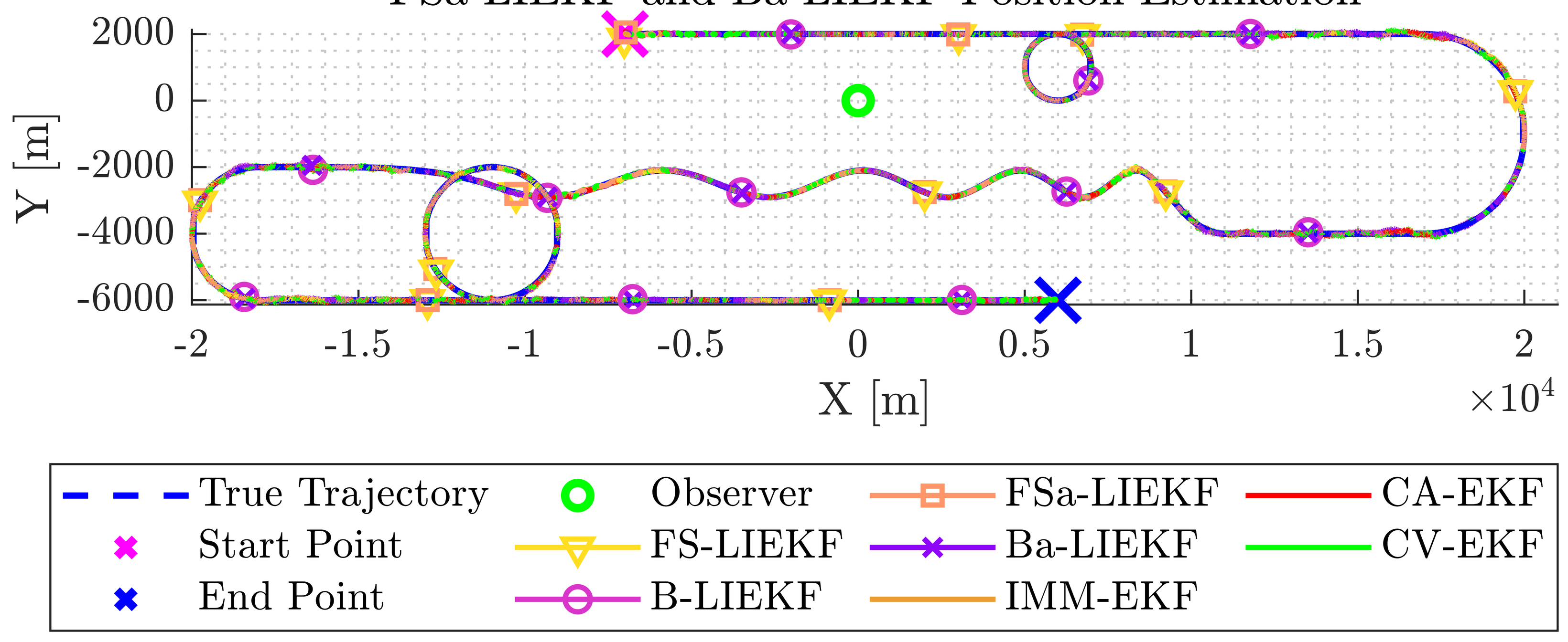

Figure 2: Sample simulation of a manoeuvring target that undergoes tangential accelerations.

Both the FSa-LIEKF and Ba-LIEKF estimate the norm acceleration a of the target along with the remaining curvature $\kappa, \kappa_1, \kappa_2$ and torsion τ parameters.

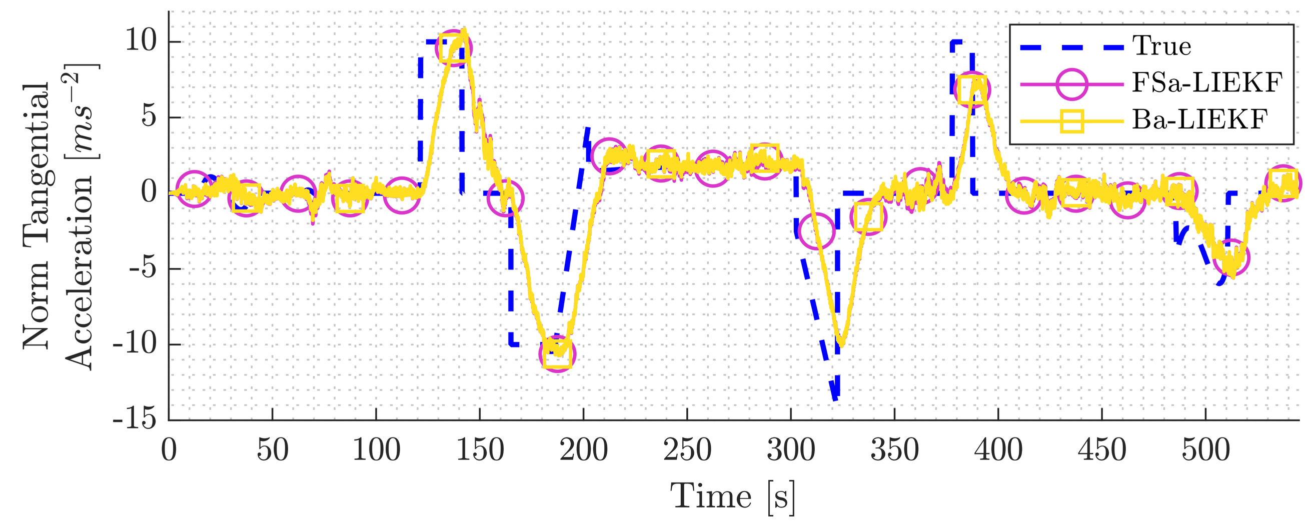

Figure 3: Norm Tangential Acceleration Estimation of the Frenet-Serret and Bishop LIEKFs.

Figure 4 shows the FSa-LIEKF and Ba-LIEKF provide more responsive estimation of the target velocity due to the additional integrator in the process model. More noise is present during trajectory segments with zero acceleration.

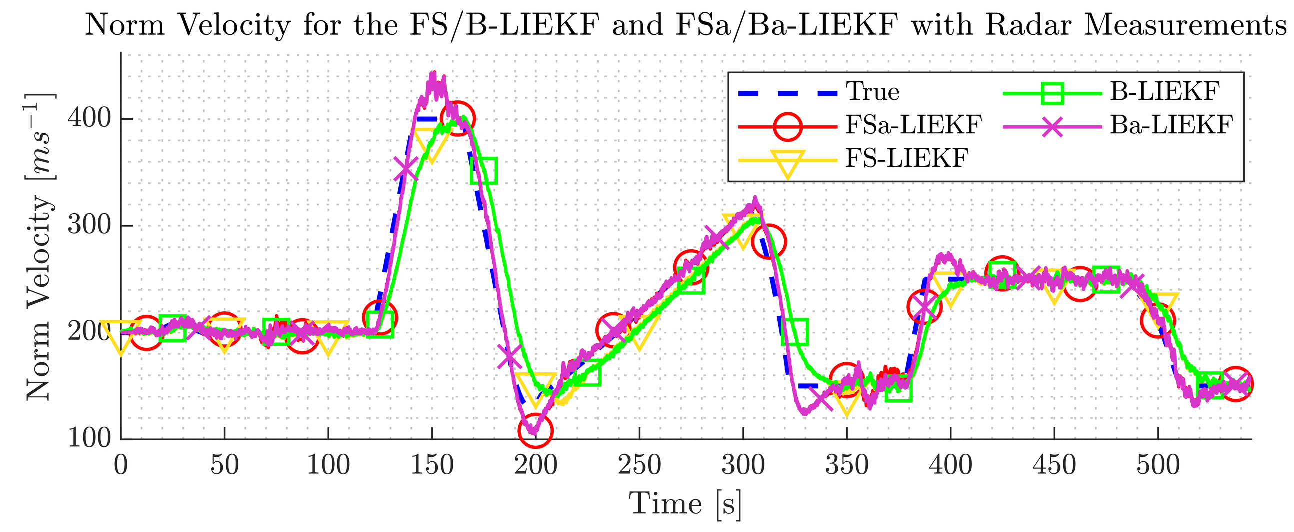

Figure 4: Norm Velocity Estimation of the Frenet-Serret and Bishop LIEKFs.

Conclusions

- FSa-LIEKF and Ba-LIEKF are shown to accurately estimate accelerating trajectories with the detriment of increased noise during constant velocity components.
- The addition of the norm acceleration in the $\hat{\zeta}_t$ component of the state allows the Frenet-Serret and Bishop kinematic models to account for a wider range of target motion compared with the FS-LIEKF and B-LIEKF.
- As the FS-LIEKF and B-LIEKF are already robust, the main benefit would come from a multiple-model algorithm featuring the constant velocity and accelerating kinematic models.

References

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