Efficient joint surface detection and depth estimation of single-photon Lidar data using assumed density filtering

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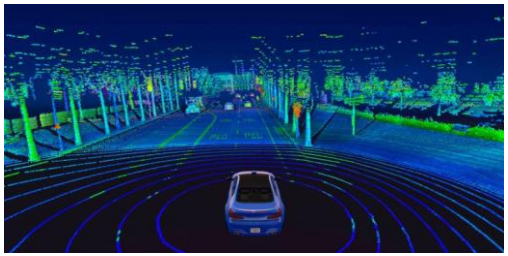
Introduction to Single-Photon Lidar (SPL)

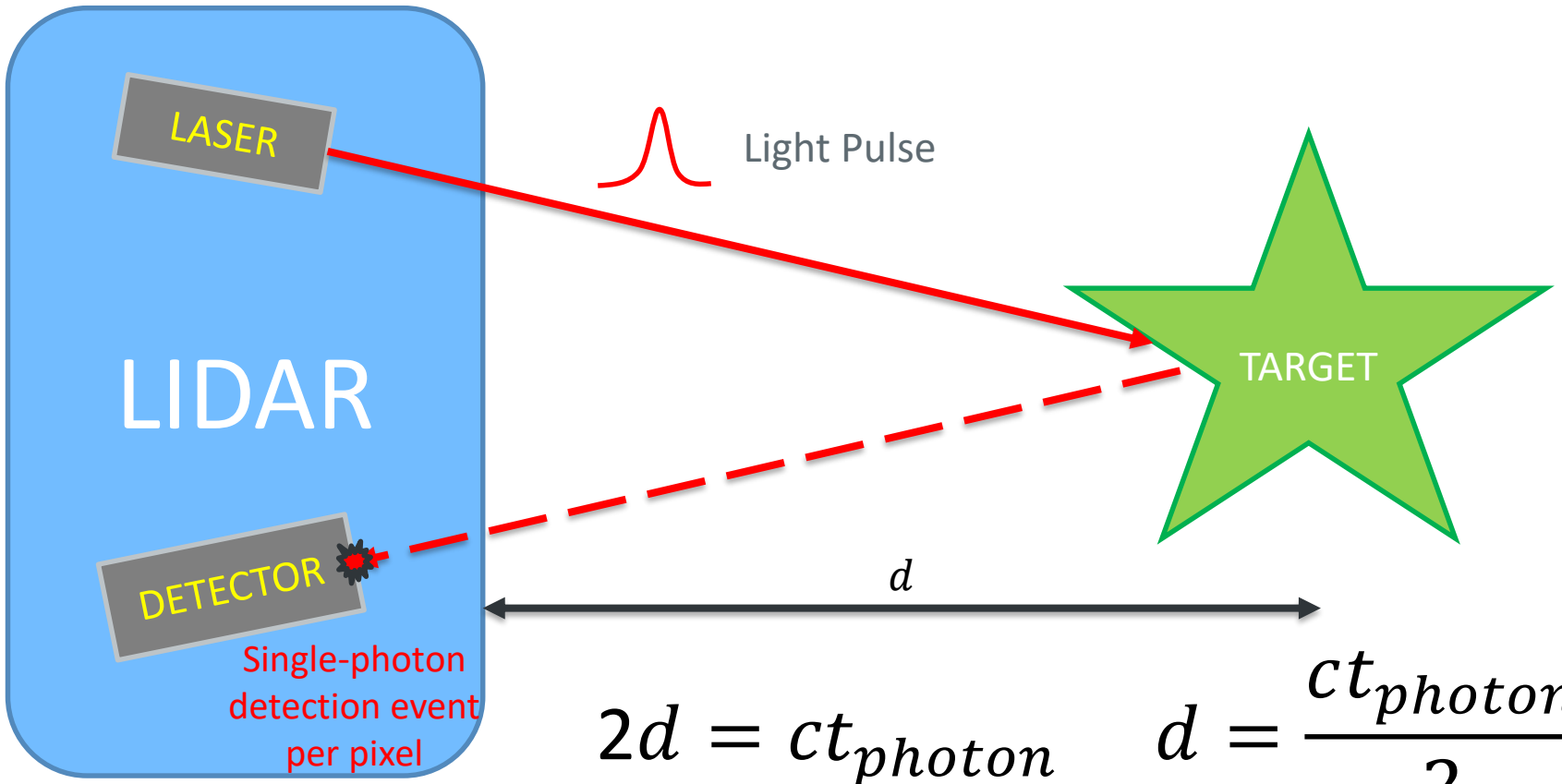
Uses and advantages of SPL



- High sensitivity: low-power, eye-safe laser sources
- picosecond timing resolution: surface-to-surface resolution at ranges up to 200 km
- Acquisition of data can be achieved at video rates or higher

Introduction to Single-Photon Lidar (SPL)

Time-Correlated Single-Photon Counting (TCSPC)



$$2d = ct_{photon} \qquad d = \frac{ct_{photon}}{2}$$

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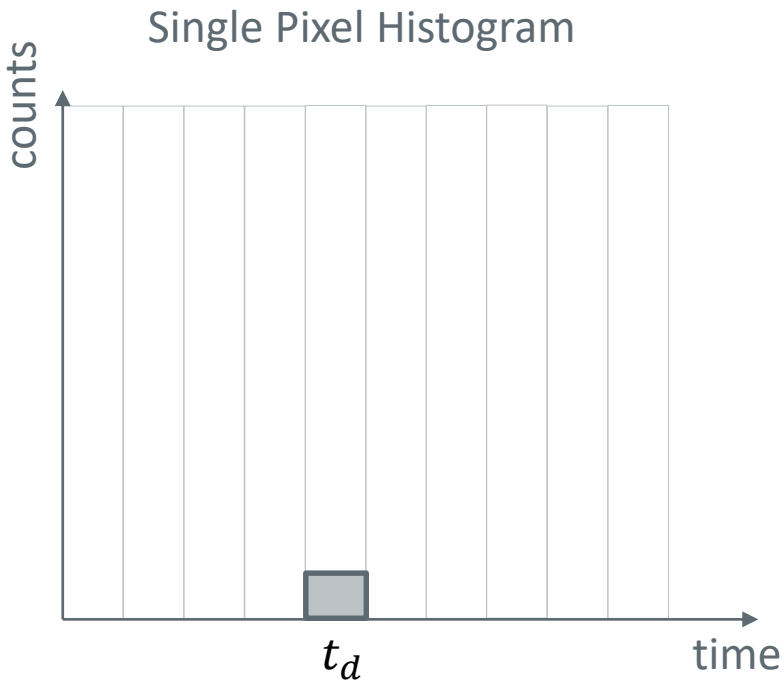

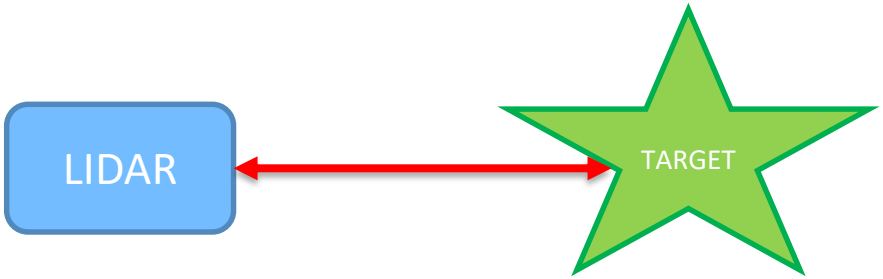

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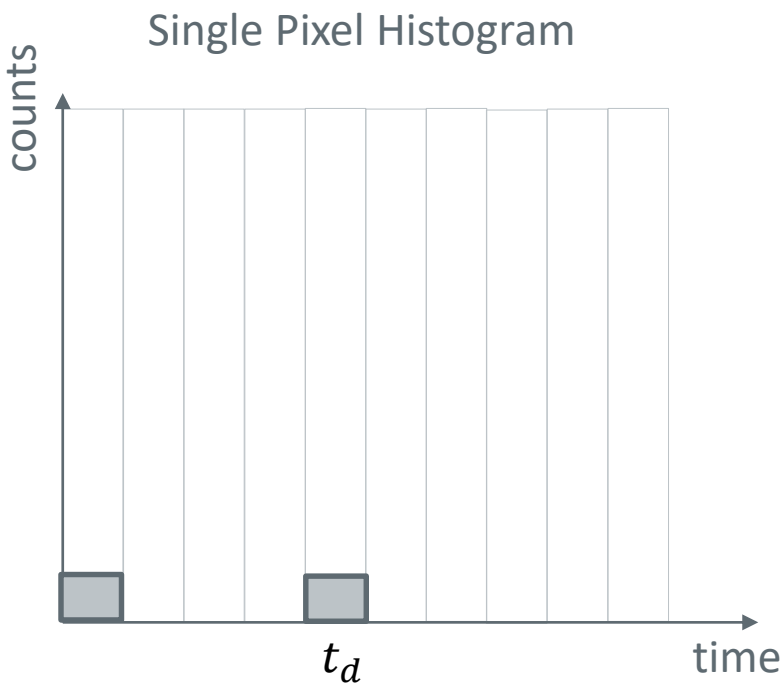

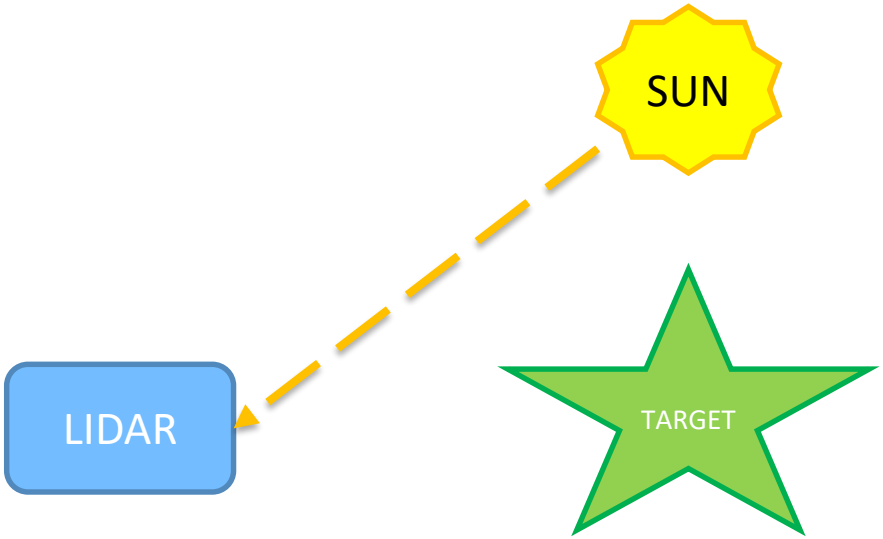

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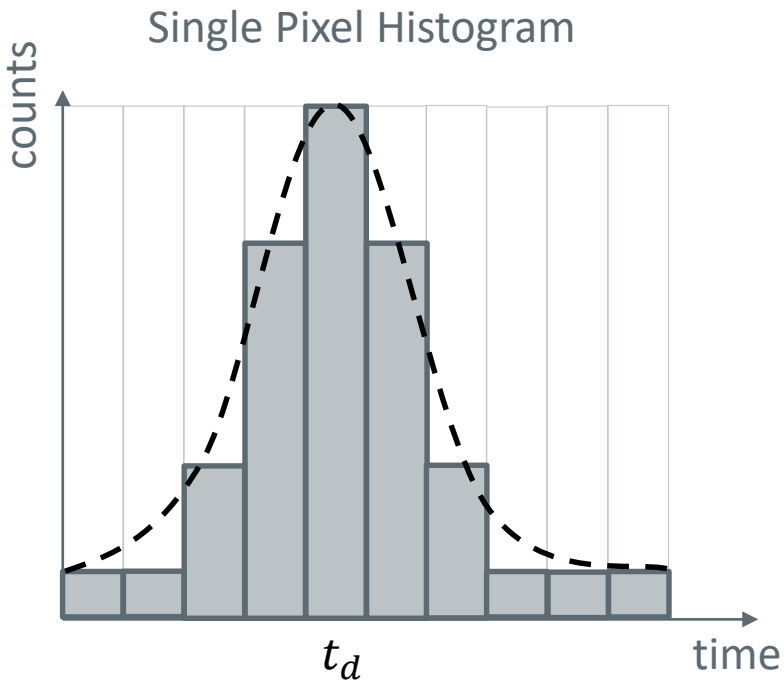
Time-Correlated Single-Photon Counting (TCSPC)

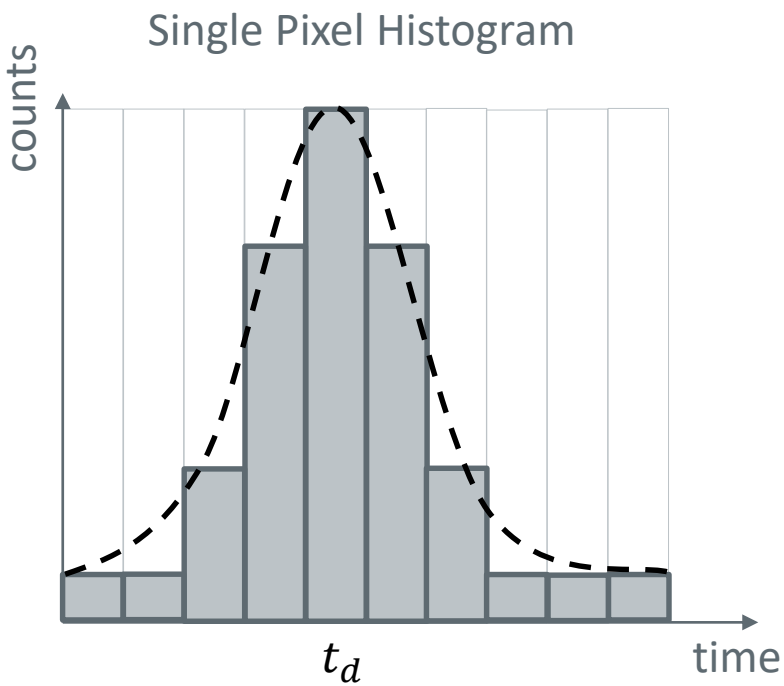


$$f(y_k|d,\mathcal{M})$$

- y_k : Single photon ToF, t_{photon}
- d : Target depth
- $\mathcal{M}$: Other model parameters

Introduction to Single-Photon Lidar (SPL)

Time-Correlated Single-Photon Counting (TCSPC)

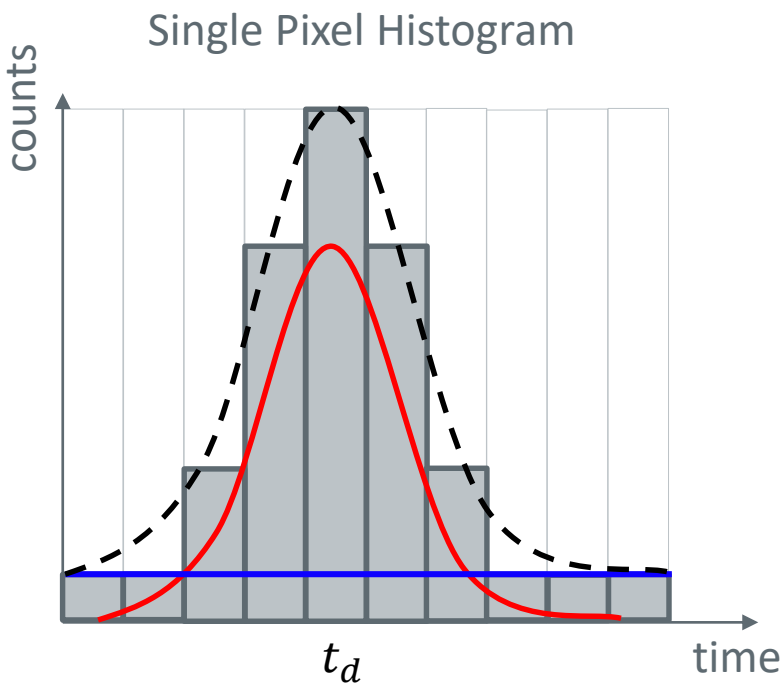


$$f(\boldsymbol{y}|d,\mathcal{M}) = \prod_{k=1}^{K} f(y_{\boldsymbol{k}}|d,\mathcal{M})$$

- y_k : Single photon ToF, t_{photon}
- d : Target depth
- $\mathcal{M}$: Other model parameters
- $\boldsymbol{y} = \{\boldsymbol{y_k}\}_{\boldsymbol{k=1}}^{\boldsymbol{K}}$: Set of K photon ToFs

Introduction to Single-Photon Lidar (SPL)

Time-Correlated Single-Photon Counting (TCSPC)



$$f(y_k|d,\omega) = \omega h_0\left(y_k - \frac{2d}{c}\right) + (1-\omega)\mathcal{V}_{[0,T]}$$

- ω : Probability of a target photon
- h_0 : Normalised IRF
- $\mathcal{V}$: Background distribution

Bayesian approach to depth estimation

$$P(x|y) = \frac{P(y|x)P(x)}{P(y)}$$

Bayes Theorem

- x, y : Events - x : what we're interested in (not defined yet)
  y : what we observe
- $P(x|y)$: Conditional distribution of x given y
- $P(y|x)$: Conditional distribution of y given x
- $P(x), P(y)$: Marginal distribution of x and y

Bayesian approach to depth estimation

$$f(d|\boldsymbol{y},\omega) = \frac{f(\boldsymbol{y}|d,\omega)f(d)}{f(\boldsymbol{y}|\,\omega)}$$

- $f(d|\boldsymbol{y},\omega)$: Posterior distribution of d
- $f(\boldsymbol{y}|d,\omega)$: Likelihood of $\boldsymbol{y}$
- $f(d)$: Prior distribution of d
- $f(\boldsymbol{y}|\,\omega)$: Marginal likelihood of $\boldsymbol{y}$

Bayesian approach to depth estimation

$$f(d|\boldsymbol{y},\omega) = \frac{f(\boldsymbol{y}|d,\omega)f(d)}{\int f(\boldsymbol{y}|d,\omega)f(d)\,\mathrm{d}d}$$

- $f(d|\boldsymbol{y},\omega)$: Posterior distribution of d
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Bayesian approach to depth estimation

$$f(d|\boldsymbol{y},\omega) = \frac{f(\boldsymbol{y}|d,\omega)f(d)}{\int f(\boldsymbol{y}|d,\omega)f(d)\,\mathrm{d}d}$$

$$f(d|\boldsymbol{y}) = \int f(d|\boldsymbol{y},\omega)\, f(\omega|\boldsymbol{y})\,\mathrm{d}\omega$$

Issues With Prior Art

Numerical approximation of integral w.r.t. ω for any value of d

INTRACTABLE - COMPUTATIONALLY INTENSIVE!

$$f(d|\boldsymbol{y}) = \int f(d|\boldsymbol{y}, \omega)\, f(\omega|\boldsymbol{y})\, \mathrm{d}\omega$$

Issues With Prior Art

Numerical approximation of integral w.r.t. ω for any value of d

RESOLVED! – SSPD 2021

$$f(d|\boldsymbol{y}) = \sum_{m=1}^{M} f(d|\boldsymbol{y}, \omega_m)\, f(\omega_m|\boldsymbol{y})$$

$$\omega = \{\omega_1, \dots, \omega_m\}$$

Issues With Prior Art

- Processing large histogram data volumes over long temporal sequences results in undesirable costs in memory requirement and computational time

- Using the whole set $\boldsymbol{y} = \{y_k\}_{k=1}^{K}$ at once prevents methods application for real-time depth estimation

Proposed Method

Propose a novel, pixel-wise, online processing method for joint surface and depth estimation from single-photon Lidar data, WITHOUT THE REQUIREMENT OF ToF HISTOGRAMS, by combining previous work on depth estimation using ensemble estimators and the online estimation strategy of …

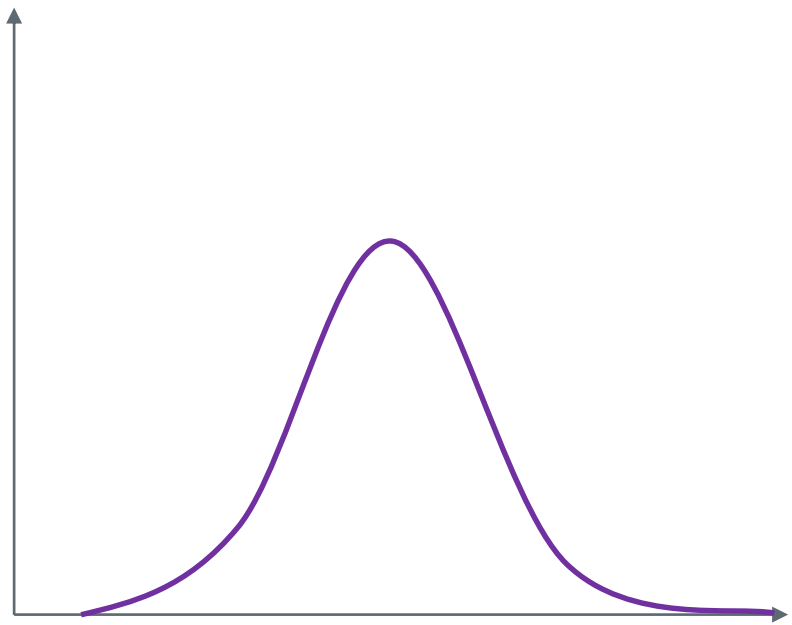
ASSUMED DENSITY FILTERING

Proposed Method

Assumed Density Filtering (ADF)

Estimate updates after every detection!

$$q^0(d) \propto \mathcal{N}\left(\mu_d^{(0)}, \left(\sigma_d^{(0)}\right)^2\right)$$



Proposed Method

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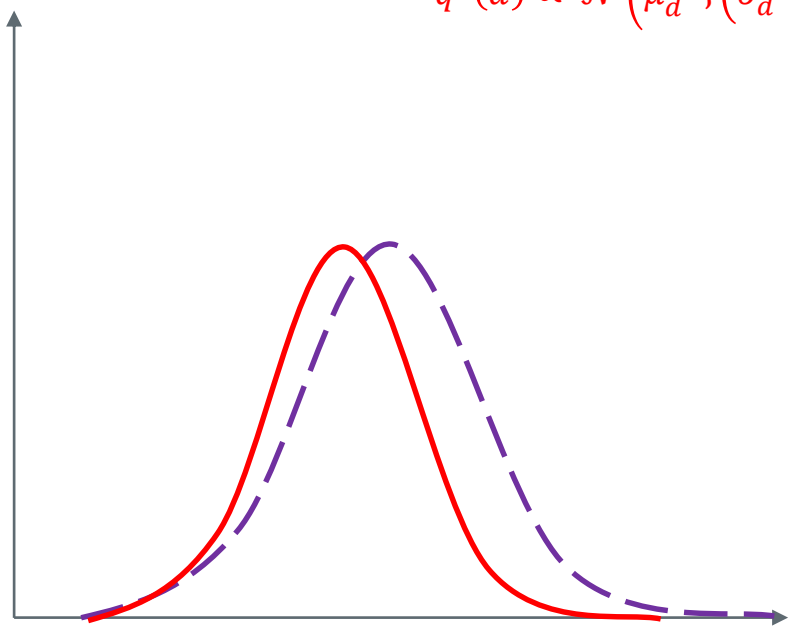

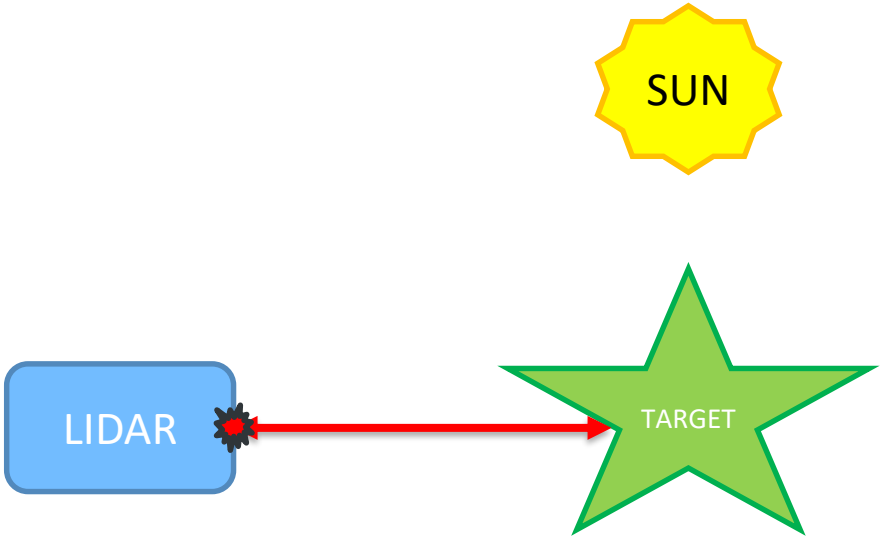

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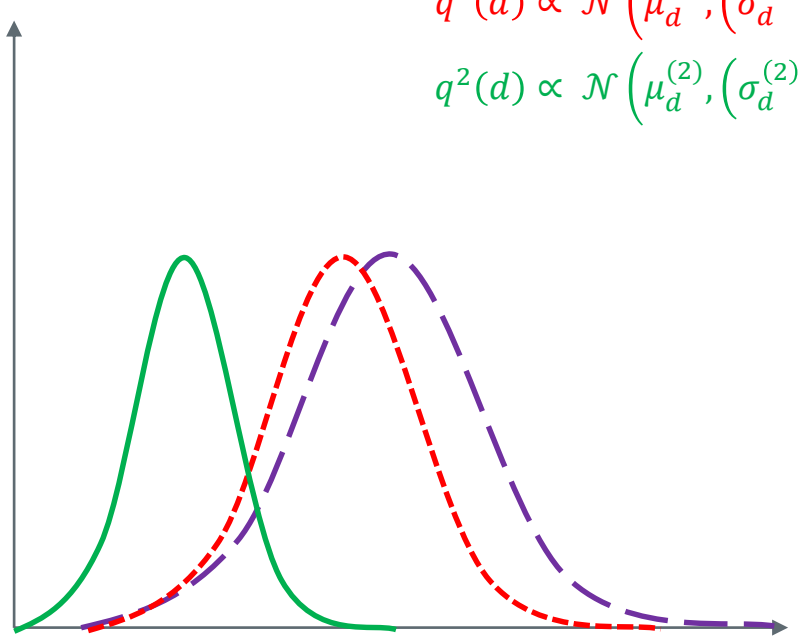
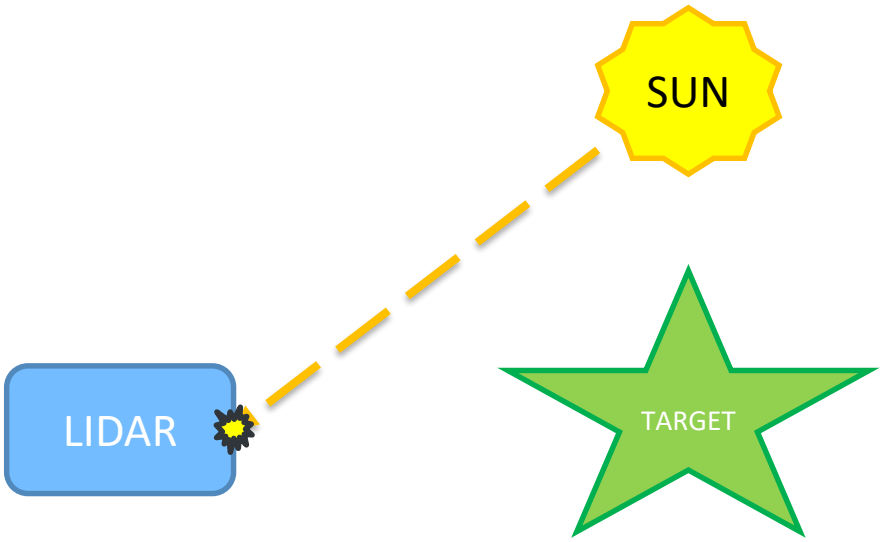
$$q^2(d) \propto \mathcal{N}\left(\mu_d^{(2)}, \left(\sigma_d^{(2)}\right)^2\right)$$





Proposed Method

Assumed Density Filtering (ADF)

Estimate updates after every detection!

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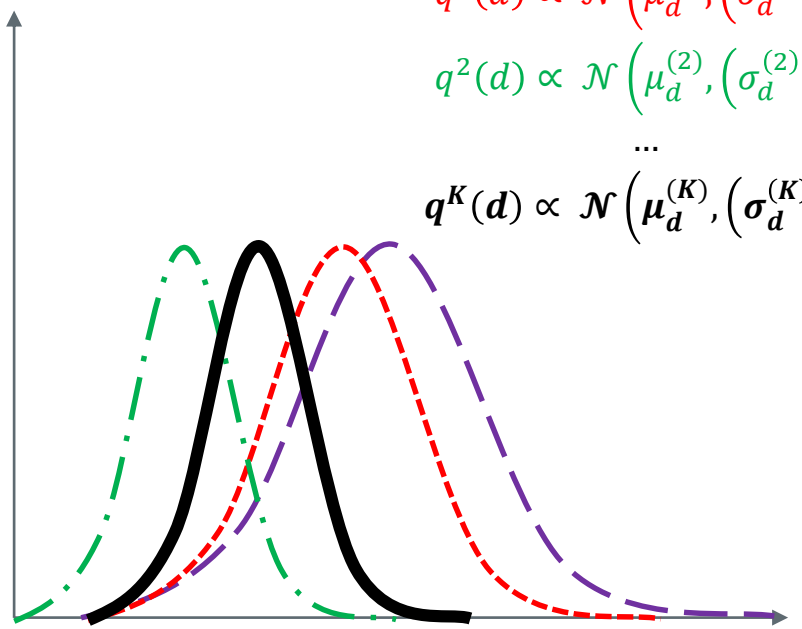
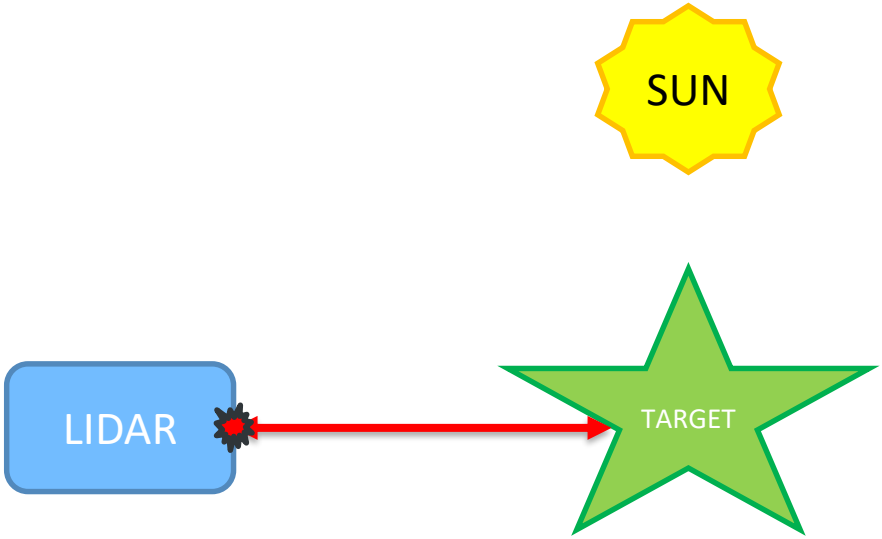
$$q^1(d) \propto \mathcal{N}\left(\mu_d^{(1)}, \left(\sigma_d^{(1)}\right)^2\right)$$

$$q^2(d) \propto \mathcal{N}\left(\mu_d^{(2)}, \left(\sigma_d^{(2)}\right)^2\right)$$

$$\ldots$$

$$\boldsymbol{q^K(d) \propto \mathcal{N}\left(\mu_d^{(K)}, \left(\sigma_d^{(K)}\right)^2\right)}$$





Proposed Method

Assumed Density Filtering (ADF)

$$\arg\min_{q^{(k)}(d)} KL\left(p^{(k)}(d|y_k,\omega) \mid q^{(k)}(d)\right)$$

Kullback-Leibler (KL) divergences

$$p^{(k)}(d|y_k,\omega) = \frac{f(y_k \mid d,\omega)q^{(k-1)}(d)}{\int f(y_k|d,\omega)q^{(k-1)}(d)\mathrm{d}d}$$

MOMENT MATCHING

$$E_q[d] = E_p[d]$$

$$E_q[d^2] = E_p[d^2]$$

Proposed Method

Assumed Density Filtering (ADF)

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MOMENT MATCHING

$$\mu_d^{(k)} = E_{p^{(k)}}[d]$$

$$\left(\sigma_d^{(k)}\right)^2 = E_{p^{(k)}}[d^2] - \left(E_{p^{(k)}}[d]\right)^2$$

Proposed Method

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$$p^{(k)}(d|y_k,\omega) = \frac{f(y_k \mid d,\omega)q^{(k-1)}(d)}{\int f(y_k|d,\omega)q^{(k-1)}(d)\mathrm{d}d}$$

$$Z_k = \int f(y_k|d,\omega)q^{(k-1)}(d)\mathrm{d}d$$

$$\prod_{k=1}^{K} Z_k \approx \int f(y|d,\omega)f(d)\,\mathrm{d}d$$

Proposed Method

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$$Z_k = \int f(y_k|d,\omega)q^{(k-1)}(d)\mathrm{d}d$$

$$\prod_{k=1}^{K} Z_k \approx f(y|\,\omega)$$

Proposed Method

Assumed Density Filtering (ADF)

$$\arg\min_{q^{(k)}(d)} KL\left(p^{(k)}(d|y_k,\omega) \mid q^{(k)}(d)\right)$$

Kullback-Leibler (KL) divergences

$$p^{(k)}(d|y_k,\omega) = \frac{f(y_k \mid d,\omega)q^{(k-1)}(d)}{\int f(y_k|d,\omega)q^{(k-1)}(d)\mathrm{d}d}$$

$$Z_k = \int f(y_k|d,\omega)q^{(k-1)}(d)\mathrm{d}d$$

$$\prod_{k=1}^{K} Z_k = s(\omega)$$

Proposed Method

Applying ADF to previous work

$$f(\omega_m|\boldsymbol{y}) = \frac{f(\boldsymbol{y}|\omega_m)f(\omega_m)}{\sum_{m=1}^{M} f(\boldsymbol{y}|\omega_m)f(\omega_m)}$$

Bayes Theorem

$$\omega = \{\omega_1, \dots, \omega_m\}$$

Proposed Method

Applying ADF to previous work

$$f(\omega_m|\boldsymbol{y}) \approx \frac{s(\omega_m)f(\omega_m)}{\sum_{m=1}^{M} s(\omega_m)f(\omega_m)}$$

Bayes Theorem

$$\omega = \{\omega_1, \dots, \omega_m\}$$

Proposed Method

Applying ADF to previous work

$$f(d|\boldsymbol{y}) = \sum_{m=1}^{M} f(d|\boldsymbol{y}, \omega_m)\, f(\omega_m|\boldsymbol{y})$$

Mixture of M Distributions

$$\omega = \{\omega_1, \dots, \omega_m\}$$

Proposed Method

Applying ADF to previous work

$$\bar{\mu} = \sum_{m=1}^{M} f(\omega_m|\boldsymbol{y})\, \mu_d(\omega_m)$$

$$\bar{\sigma}^2 = \left(\sum_{m=1}^{M} f(\omega_m|\boldsymbol{y})\, (\sigma_d(\omega_m)^2 + \mu_d(\omega_m)^2) \right) - \bar{\mu}^2$$

Proposed Method

Applying ADF to previous work

$$\bar{\mu} = \sum_{m=1}^{M} f(\omega_m|\boldsymbol{y})\, \mu_d(\omega_m)$$

(μ_d, σ_d^2)
Obtained from final Normal distribution estimation from ADF for $\omega = \omega_m$

$$\bar{\sigma}^2 = \left(\sum_{m=1}^{M} f(\omega_m|\boldsymbol{y})\, (\sigma_d(\omega_m)^2 + \mu_d(\omega_m)^2) \right) - \bar{\mu}^2$$

Proposed Method

Applying ADF to previous work

Can also determine if a surface is present in pixel!

Define ω_0, δ such that $\forall\ \omega_m > \omega_0$, where $\omega_m \in \omega$

$$\sum_{\omega_m > \omega_0} f(\omega_m | \boldsymbol{y}) \geq \delta$$

Likely Surface Estimate!

Proposed Method

Applying ADF to previous work

Can also determine if a surface is present in pixel!

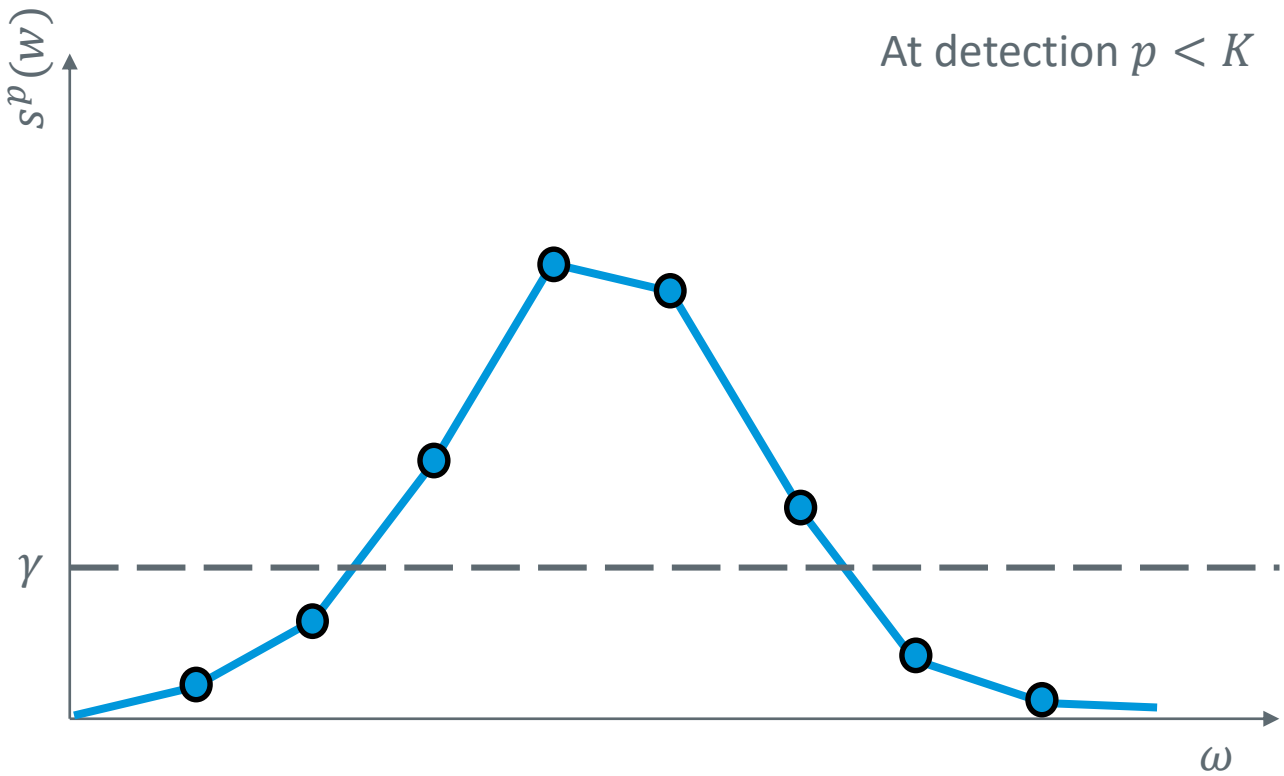
Define ω_0, δ such that $\forall\ \omega_m > \omega_0$, where $\omega_m \in \omega$

$$\sum_{\omega_m > \omega_0} f(\omega_m | \boldsymbol{y}) < \delta$$

Likely Noise!

Proposed Method

Reduction of discrete ω parameter list



Proposed Method

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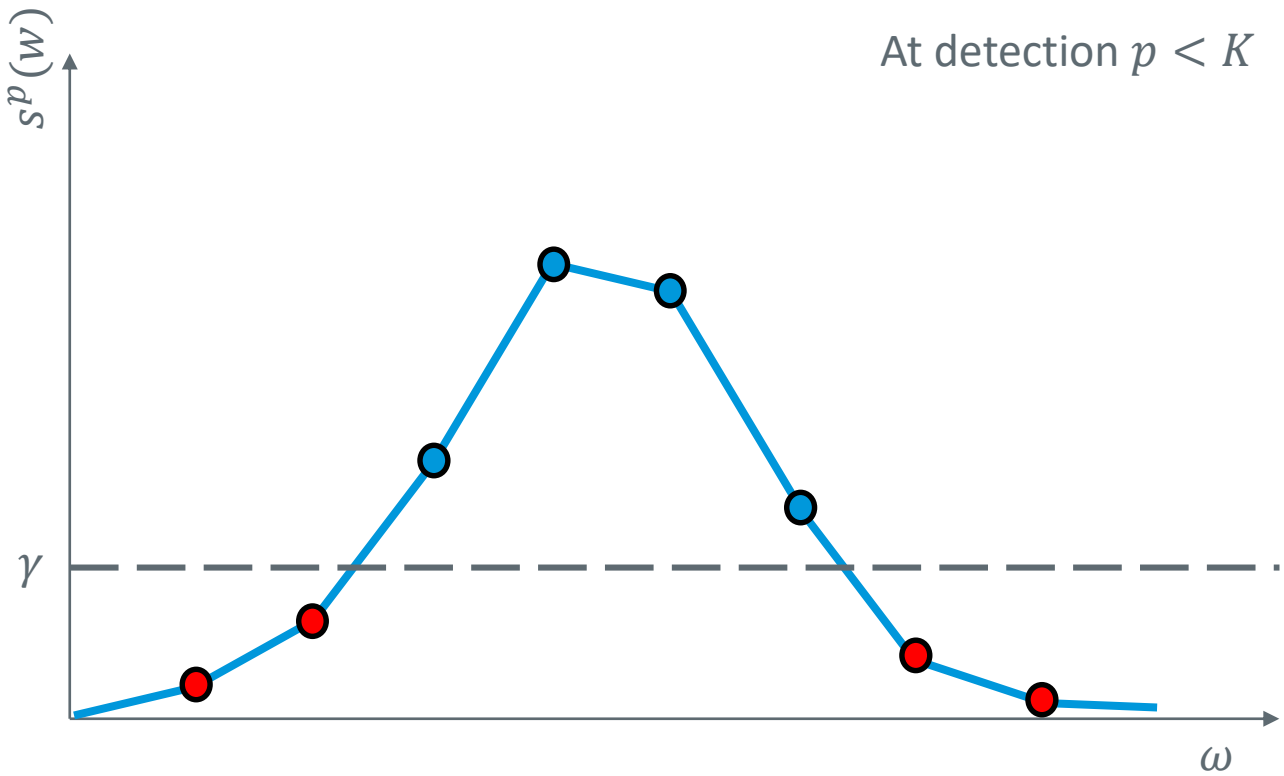

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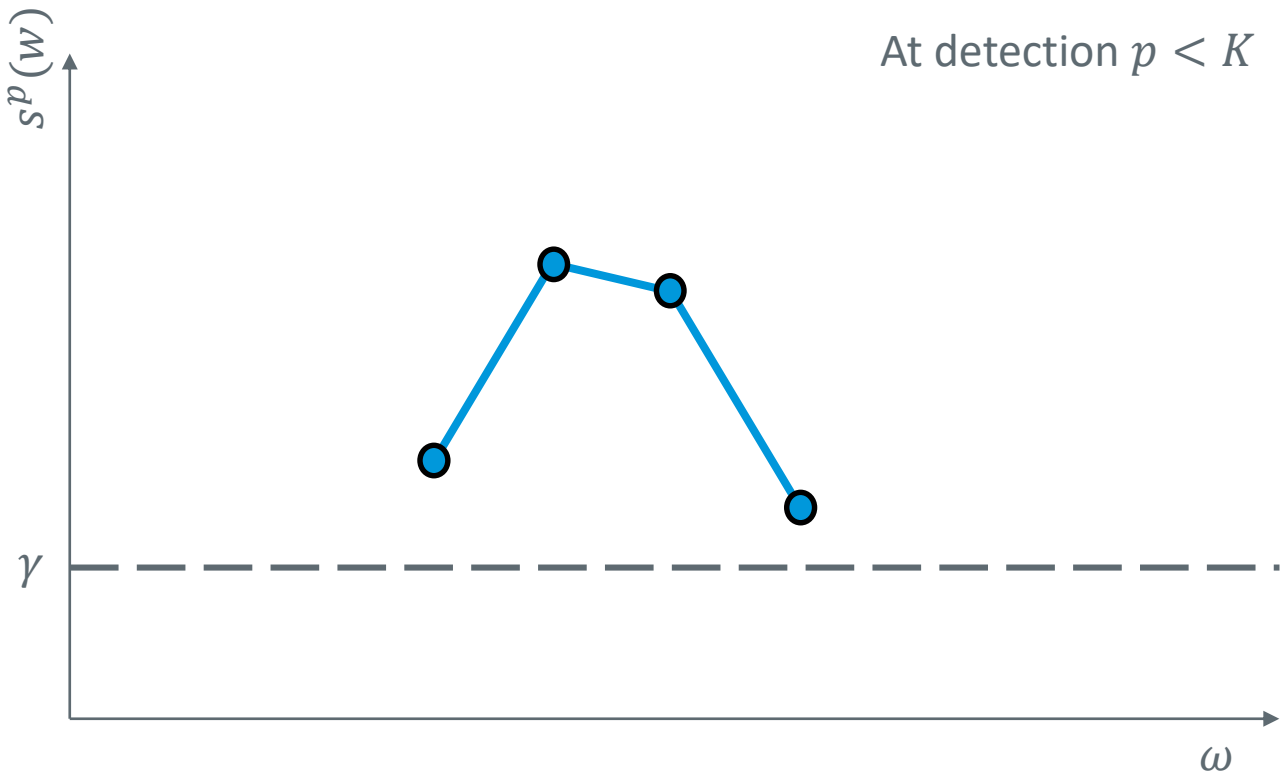


Results

Synthetic Data

	Depth		w	Time (s)
	$\bar{\mu}(m)$	$\bar{\sigma}^2(m^2)$		
ADF basic ($M = 20$)	17.99 (7.67e-3)	5.18e-5 (4.71e-6)	0.22 (0.02)	0.034
ADF basic ($M = 100$)	17.99 (7.20e-3)	5.18e-5 (4.96e-6)	0.22 (0.02)	0.133
ADF * warm-start	17.99 (7.20e-3)	5.18e-5 (7.83e-3)	0.20 (0.01)	0.194
Reduction mthd. 1 **	17.99 (7.67e-3)	5.18e-5 (7.70e-3)	0.20 (0.01)	0.141
**Reduction mthd. 2 ** **	**17.99 (7.20e-3)**	**5.18e-5 (7.87e-3)**	**0.20 (0.01)**	**0.100**
Drummond [18] ($M = 20$)	17.99 (6.24e-3)	4.03e-5 (4.60e-5)	0.20 (0.01)	0.012
Drummond [18] ($M = 100$)	17.99 (6.24e-3)	4.03e-5 (4.03e-5)	0.20 (0.01)	0.059
cross correlation	17.99 (7.67e-3)	N/A (no std.)	0.2 (w known)	0.001

ADF Methods

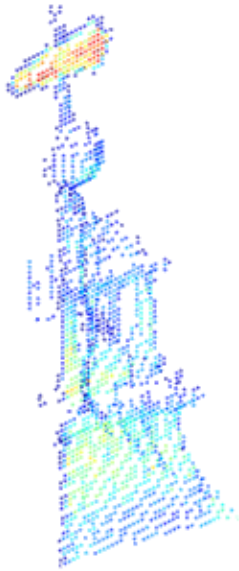
Histogram Methods

TABLE I: Comparison of the different depth and w estimates for different methods. Values in brackets represent standard deviations over 1000 results. The actual value of (d, w) is $(17.99m, 0.2)$.$^{(*)}$: this method uses $M = 100$. $^{(**)}$: these methods start with $M = 100$ and warm-start.

Times shown are the times for processing the histogram data and do not include histogram acquisition times.

Results

Real College Tower Data

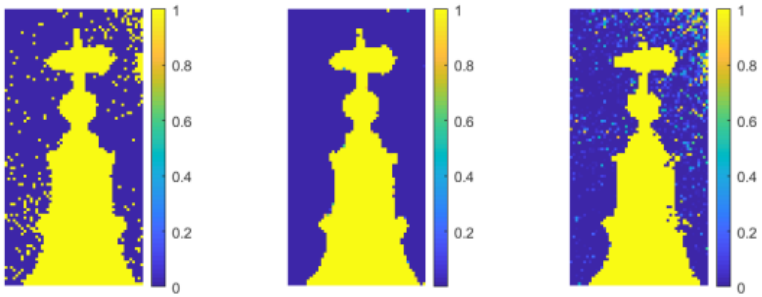


Data Provided by Leonardo UK

Range ~ 3 km

Results

Real College Tower Data

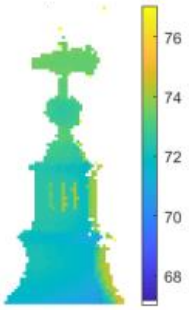


(a) Sheehan [19]

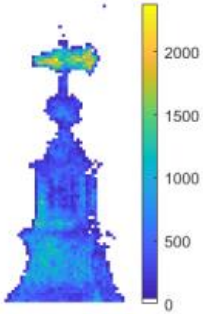
(b) Drummond $w_0 = 0.02$ [18]

(c) Proposed: $w_0 = 0.02$

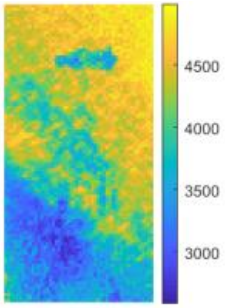
depth



intensity



background



Conclusion

- Proposed an extension to ensemble estimator method and produced satisfactory results using ADF to obtain posterior distribution profiles for final:
  - surface detection,
  - depth estimation and
  - uncertainty estimates.

- Able to further improve efficiency by eliminating values from discrete variables depending on corresponding posterior distribution profile value.

Future Work

- GPU implementation to enable reliable depth estimation and uncertainty quantification at real-time speeds.

- Plan to adapt framework to richer approximations of the posterior distributions of d, allowing for multiple surface detections per pixel.

Thanks for your attention !

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