Random Sampling for Robust Detection of Data modulated LFM Waveforms

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UDRC WP2.2 Reconfigurable signal processing

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Research topic — Joint radar and communication

- **Sixth generation** (6G) is the next generation mobile system for wireless communications technologies;
- Waveforms in 6G — Convergence of **Communications, Computing, Control, Localisation, and Sensing** (3CLS);
  - Integrated Sensing and Communication **(ISAC)** waveform;
  - **Radar** for target localisation (e.g., range & velocity);
  - **Communication** for information transmission;
  - Useful both in **defence** and **civilian** applications.

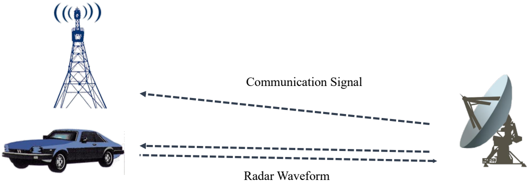


Figure: Application scenario — Joint radar and communication.

Our Research

- Research goals:
  - Investigate novel **detection methods** for modified/traditional radar waveform via signal processing;
  - Improve the **robustness** of our method in imperfect conditions.
- Research contents:
  - Traditional linear frequency modulated **(LFM)**/chirp waveform:

$$f(t) = \exp(\mathrm{j}(\pi f_l t^2 + 2\pi f_k)). \tag{1}$$

  - f_l determines how quickly **the chirp frequency** changes.
  - f_k determines **the start frequency** of the chirp signal.
  - Technique: Discrete Chirp-Fourier Transform **(DCFT)**[1].

[1] X. -g. Xia, "Discrete chirp-Fourier transform and its application to chirp rate estimation," *IEEE Transactions on Signal Processing*., vol. 48, no. 11, pp. 3122-3133, Nov. 2000.

DCFT Detection Method

- The discrete format of signal $x[n] = f(n/N)$ with N samples;
- N-**point DCFT** method is applied to $x[n]$ with the twiddle factor $W_N = \exp(-2\pi \mathrm{j}/N)$

$$X[l,k] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n] W_N^{ln^2+kn} \qquad l,k = 0,1,\ldots,N-1. \quad (2)$$

- DCFT: chirp signal ➜ **two dimensional frequency domain**;
- l represents f_l and k denotes f_k;
- $(\tilde{l}, \tilde{k})$ corresponds to the **highest** value of the matrix $X[l,k]$;
- The estimated LFM parameters $\tilde{f}_l$ and $\tilde{f}_k$ are

$$\tilde{f}_l = 2N\tilde{l}, \qquad \tilde{f}_k = \tilde{k}\,. \quad (3)$$

Simulation Example

- The LFM $f(t)$ chirp frequency $f_l = 550\,\mathrm{Hz}$ and offset frequency $f_k = 30\,\mathrm{Hz}$. In the DCFT detection method when $N = 55$.

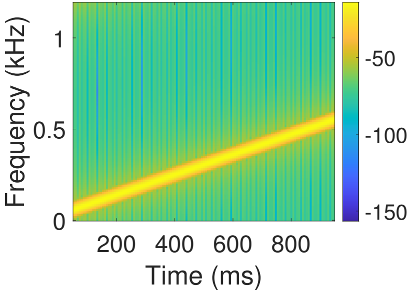


Figure: LFM $f(t)$ spectrogram.

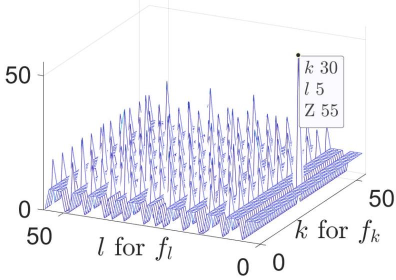


Figure: DCFT output matrix $X[l, k]$.

- Estimation Result:

$\tilde{f}_l = 2N\tilde{l} = 2 \times 55 \times 5 = 550\,\mathrm{Hz}$ $\qquad$ $\tilde{f}_k = \tilde{k} = 30\,\mathrm{Hz}.$

Coherent DCFT[2] — Customised Ranges and Resolutions

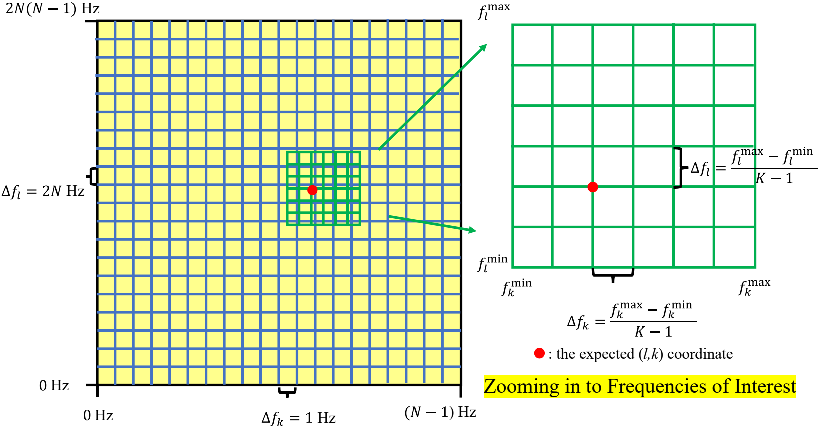


Figure: 2D Frequency Grid of DCFT (Left) and coherent-DCFT (Right).

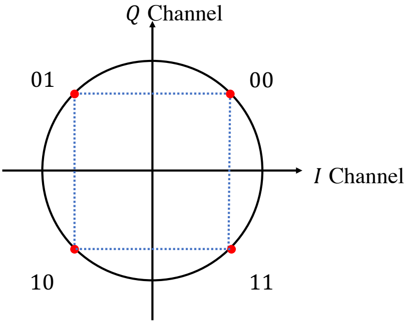
[2] K. Zhang, F. K. Coutts and J. Thompson, "Detecting LFM Parameters in Joint Communications and Radar Frequency Bands," 2021 Sensor Signal Processing for Defence Conference (SSPD), Edinburgh, United Kingdom, 2021, pp. 1-5.

Joint radar and communication (JRC)

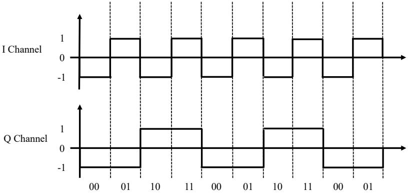
- Radar-Communication Coexistence (RCC).
  - Task: to efficiently **allocate the spectrum** for both radar waveforms and communication signals
    - Explore Spectrum Allocation; Opportunistic Spectrum Access; Interference Issues.
- Dual-function radar communication (DFRC) systems.
  - Task: use one waveform to perform radar and communication functions **simultaneously**
    - Waveform design based on **communication standard** such as orthogonal frequency-division multiplexing (OFDM);
    - Waveform designed based on **radar waveform**, e.g., LFM/chirp signal.

Data modulated LFM waveform

- Example: Quadrature Phase Shift Keying (QPSK)-LFM waveform
- Symbols Example $S_n(t)$: 00 01 10 11 00 01 10 11 00 01



(a) QPSK constellation.



(b) Example symbols.

Figure: QPSK example diagrams.

QPSK-LFM waveform

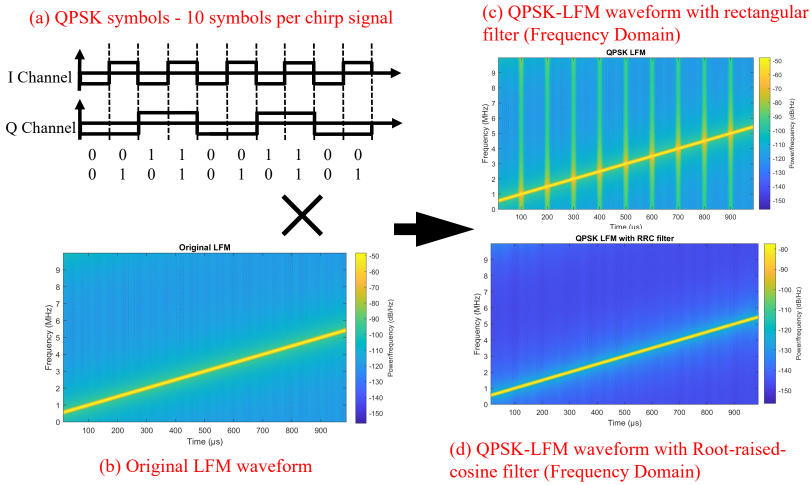


Figure: QPSK-LFM generation process.

Non-Coherent DCFT method (NC-DCFT)

- Signal: Data modulated LFM waveforms.
- Problem: **Random** phase information from different symbols.
- Impact: **Deteriorate** the coherent DCFT performance.
- Principle: Coherent DCFT works within a fixed signal phase.
- Solution — NC-DCFT process:
  - Divide received signal for each symbol;
  - Apply coherent DCFT in each segment;
  - Implement norm function to counter uncertain PSK information;
  - Summation for the final result.

Non-Coherent DCFT Diagram

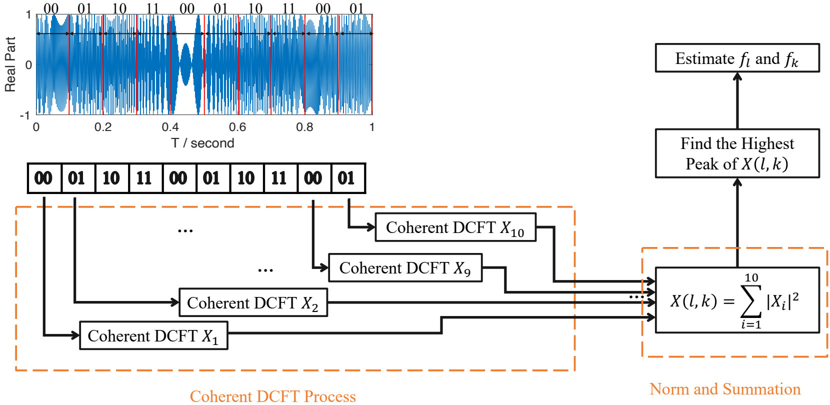


Figure: Example of non-coherent DCFT process when symbol length is **known** to be 10 symbols per chirp signal.

Key Simulation Parameters.

Name	Value/Interpretation
the length of the DCFT K	128 or 256
f_l Estimation Range $(f_l^{\min}, f_l^{\max})$	$(0, 3 \times 10^5)$ Hz
f_k Estimation Range $(f_k^{\min}, f_k^{\max})$	$(10^9, 10^{10})$ Hz
Chirp period T	1 ms
Number of Symbols per chirp	50 (known)
Sample Frequency f_s	10^7 Hz
Noise Type $w(t)$	Additive White Gaussian Noise
Signal to Noise Ratio (SNR)	$[-30 \text{ dB}, 0 \text{ dB}]$
Performance Metric	Normalised Mean Square Error*

*Normalised Mean Square Error (NMSE):

$J_{\text{NMSE}} = \frac{\mathbf{d}\mathbf{d}^{\text{T}}}{\mathbf{g}\mathbf{g}^{\text{T}}}$, $\mathbf{g}$ consists of ground truth g_l or g_k in each Monte Carlo runs and for each element in $\mathbf{d}$ is $(\tilde{f}_l - g_l)$.

Simulation results

DCFT vs Coherent DCFT for LFM

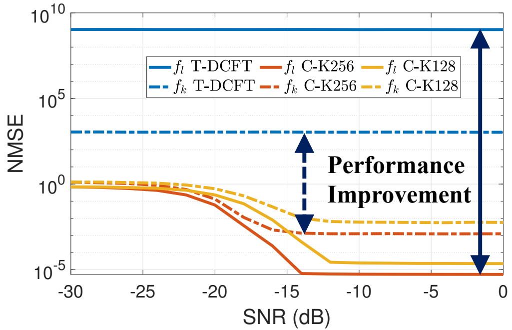


Figure: Comparison of traditional **DCFT** and **coherent DCFT** results.

- Coherent DCFT methods outperform the DCFT method with the customised range;
- Higher value of K improves the performance of coherent DCFT.

Coherent DCFT vs Non-coherent DCFT for QPSK-LFM

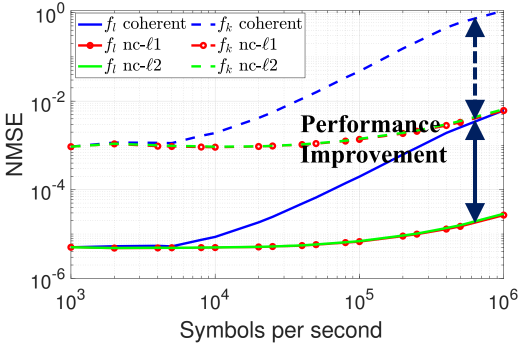


Figure: Comparison of **coherent** DCFT and **Non-coherent DCFT** results in different symbol rates.

- Non-coherent DCFT performs well when the symbol rate increasing;
- ℓ_1 norm performs similarly as ℓ_2 norm.

Evolution of our Detection Algorithms

- Our previous research on the detection of QPSK-LFM waveform:
  - DCFT detection method — proposed for **LFM signal**;
  - **Coherent DCFT** method — updated on DCFT detection for higher performance[3];
  - **Non-coherent DCFT** method — modified on the coherent DCFT for data modulated LFM waveform[3].
- Our current research — blind estimation & improve robustness.
  - **Blind task**: Estimate the symbol rate for QPSK-LFM waveforms;
  - **Robustness scenario**: Imperfect synchronisation with time errors;
  - Solutions: **Envelope method** and **random sampling method**.

[3] K. Zhang, F. K. Coutts and J. Thompson, "Non-Coherent Discrete Chirp Fourier Transform for Modulated LFM Parameter Estimation," 2022 Sensor Signal Processing for Defence Conference (SSPD), London, United Kingdom, 2022, pp. 1-5.

Comparison Table for Different DCFT Methods.

	DCFT	Coherent DCFT	Non-coherent DCFT
The length	Fixed length N	Customised K	Customised K
$(f_l^{\min}, f_l^{\max})$	Fixed	Customised	Customised
$(f_k^{\min}, f_k^{\max})$	Fixed	Customised	Customised
Resolutions Δf_l Δf_k	Coarse	Fine	Fine
Application	**LFM**	**LFM**	**QPSK-LFM** with known symbol rate

Modifications on NC-DCFT method

- Envelope method — **Blind** symbol rate estimation.
- Envelope-NC-DCFT: Envelope method 1st and then NC-DCFT applied.

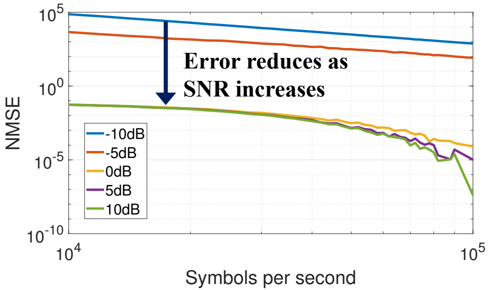


Figure: Performance of envelope method for the symbol rate estimation result.

Envelope method processing:

- **Square** the modulus of the received signal;
- Apply the Fast Fourier Transform (**FFT**);
- Select the **largest** magnitude in the FFT bin;
- Corresponds to the symbol rate.

Robustness improvement

Application scenario - Imperfect synchronisation with timing errors

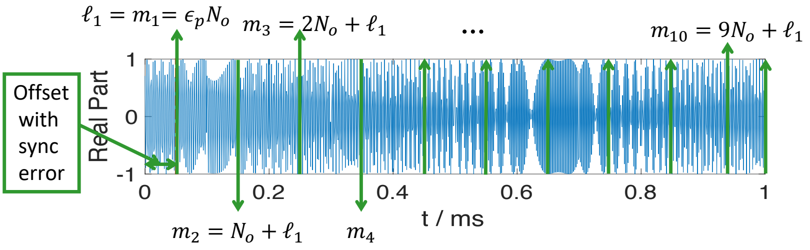


Figure: Diagram of imperfect synchronisation scenario.

- Application — Mismatched sync and ℓ_1 length samples **lost**.
- ϵ_p the ratio of ℓ_1 and the oversampling rate N_o.
- Negative Impact — Non-coherent DCFT fails due to **phase information cancellation**.

Performance of NC-DCFT under imperfect synchronisation

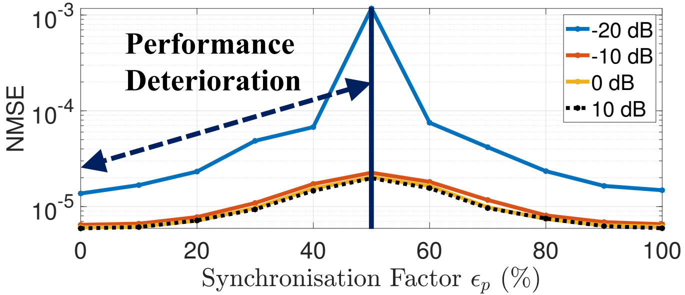


Figure: Performance of NC-DCFT for f_l estimation.

- Imperfect synchronisation increases the **error** of estimation.
- The **worst** performance at 50% and gradually improves until 100%.

Solutions — Random sampling method

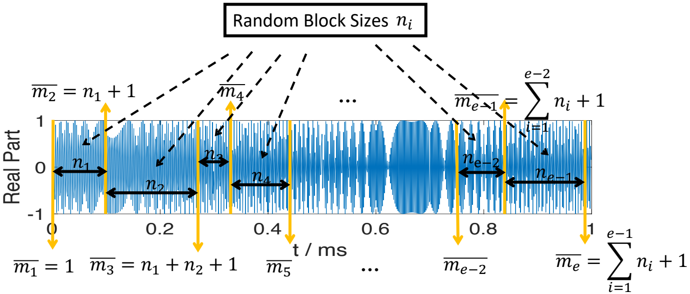


Figure: Diagram of random sampling method via different points $\overline{m_e}$.

- **Random length** n_i for ith coherent DCFT.
- Avoid **phase cancellation** through variable block sizes.
- The ground truth symbol rate is **unknown**.
- RS-NC-DCFT: Random sampling method is combined with the NC-DCFT.

Simulations — Random sampling range

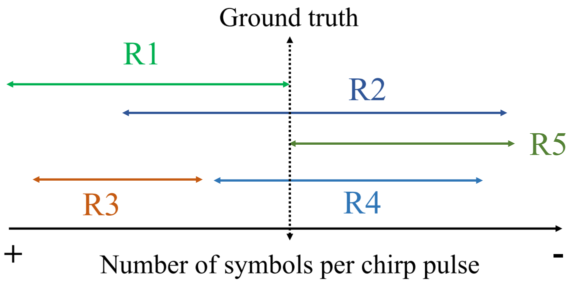


Figure: Different simulation ranges R_i for the symbol rate.

- R1 and R3 are **larger** than ground truth symbol rate.
- R2 and R4 **straddle** the ground truth symbol rate.
- R5 are **less** than ground truth symbol rate.

Simulations on Random Sampling

Perfect Time Synchronisation

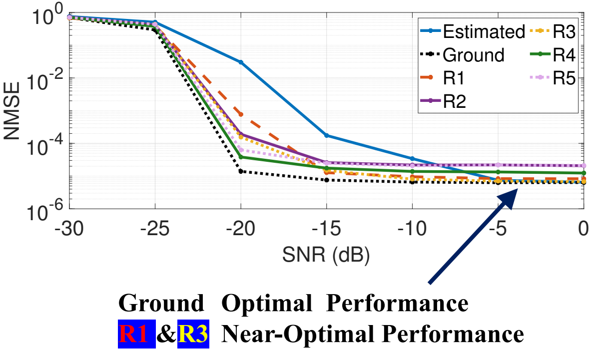
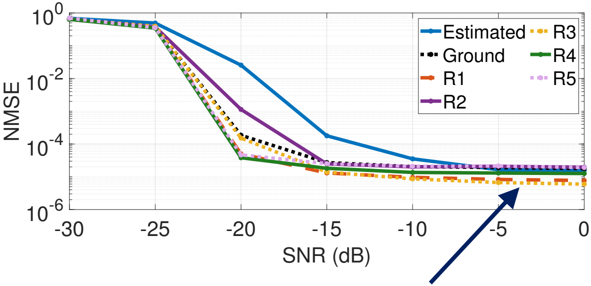


Figure: Performance comparison of chirp rate estimation in different conditions.

- Blind test method works well in higher SNR;
- R1 and R3 (**larger** than ground truth) performs similarly as the ground truth;
- Random sampling method is the viable option when ground truth is unknown.

Imperfect synchronisation



R3 Optimal Performance
R1 Near-Optimal Performance

Figure: Performance comparison of chirp rate estimation when $\epsilon_p = 50\%$.

- R1 and R3 (**larger** than ground truth) performs better than the ground truth;
- Random sampling method is a practical method to handle imperfect synchronisation.

Conclusions

- Detection method is proposed for joint radar and communication.
- The NC-DCFT method works when both the symbol rate is known and time synchronisation is performing well.
- When the envelope method is applied, the Envelope-NC-DCFT performs well in situations with high SNRs.
- With the random sampling technique, the RS-NC-DCFT is an alternative strategy for the NC-DCFT when the symbol rate is unknown.
- The RS-NC-DCFT demonstrates superior performance over the NC-DCFT to combat time synchronisation problems.

Working conditions for different DCFT relevant methods.

	Unknown Symbol rate	Imperfect Synchronisation	Waveform Type
DCFT	✗	✗	LFM
Coherent DCFT	✗	✗	LFM
Non-Coherent DCFT	✗	✗	QPSK-LFM
Envelope-NC-DCFT	✔	✗	QPSK-LFM
RS-NC-DCFT	✔	✔	QPSK-LFM