Generalised Sequential Matrix Diagonalisation (GSMD)

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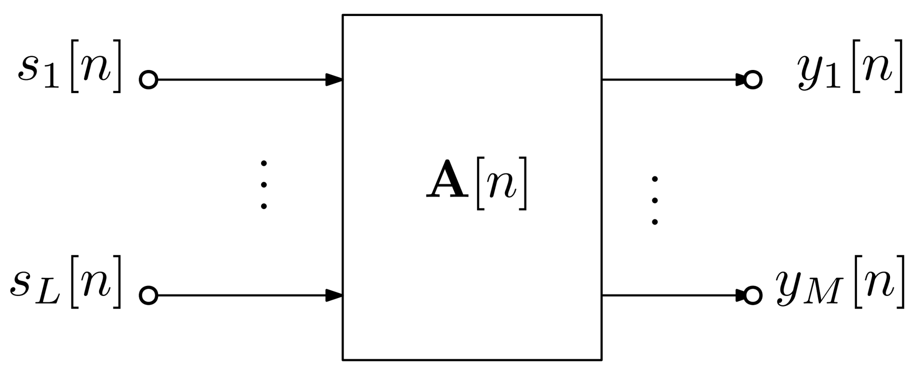
Abstract

We present an innovative extension of the sequential matrix diagonalization (SMD) technique, a para-Hermitian polynomial matrix iterative polynomial eigenvalue decomposition (PEVD) approach. This extension, termed as Generalized SMD (GSMD), broadens the application scope to encompass the general polynomial matrix singular value decomposition.

Motivation

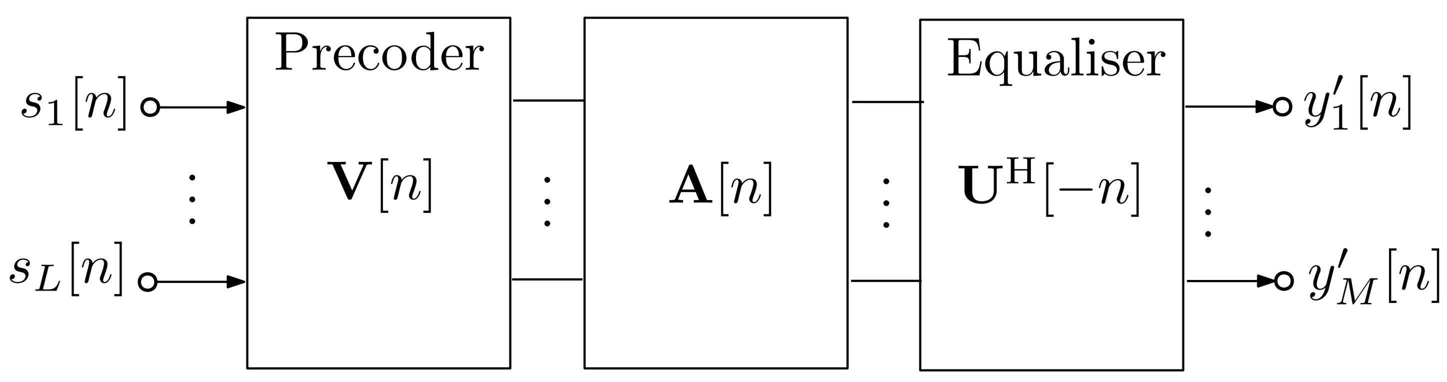
Consider a broadband $M \times L$ MIMO system



where $s_\ell[n], \ell = 1, \ldots, L$ sources. The elements of the matrix $\mathbf{A}[n]$ are time-sequences instead of complex-gain factors. Its $z-$transform $\boldsymbol{A}(z)$ will be a matrix of transfer functions, and therefore a polynomial matrix. Such MIMO system design i.e. precoding and equalisation can be based on SVD of

$$\boldsymbol{A}(z) = \boldsymbol{U}(z)\boldsymbol{\Sigma}(z)\boldsymbol{V}^{\mathrm{P}}(z) \tag{1}$$

as



resulting $\boldsymbol{y}'(z) = \boldsymbol{\Sigma}(z)\boldsymbol{s}(z)$, where $\boldsymbol{\Sigma}(z)$ is diagonal matrix. Paraunitarity of $\boldsymbol{V}(z)$ and $\boldsymbol{U}(z)$ causes (i) no noise amplification, (ii) no change in transmit power.

Polynomial SVD Algorithms

1. Two polynomial EVDs (PEVDs)

   $\boldsymbol{V}(z) \leftarrow$PEVD$\{\boldsymbol{A}^{\mathrm{P}}(z)\boldsymbol{A}(z)\}$; $\boldsymbol{U}(z) \leftarrow$ PEVD$\{\boldsymbol{A}(z)\boldsymbol{A}^{\mathrm{P}}(z)\}$
   $\Rightarrow \boldsymbol{\Sigma}(z) = \boldsymbol{U}^{\mathrm{P}}(z)\boldsymbol{A}(z)\boldsymbol{V}(z)$

2. Iterative applications of polynomial QRD (PQRD)

   $[\boldsymbol{Q}_1(z), \boldsymbol{R}_1(z)] =$ PQRD($\boldsymbol{A}(z)$);
   $[\boldsymbol{Q}_2(z), \boldsymbol{R}_2(z)] =$ PQRD($\boldsymbol{R}_1^{\mathrm{P}}(z)$);
   $\boldsymbol{A}(z) \leftarrow \boldsymbol{R}_2^{\mathrm{P}}(z)$, $\boldsymbol{U}(z) \leftarrow \boldsymbol{U}(z)\boldsymbol{Q}_1(z)$; $\boldsymbol{V}(z) \leftarrow \boldsymbol{V}(z)\boldsymbol{Q}_2(z)$

3. Generalized second-order sequential best rotation (GSBR2): This dedicated PSVD algorithm employs SBR approach but with Kogbetliantz transformation instead of only Givens rotation.

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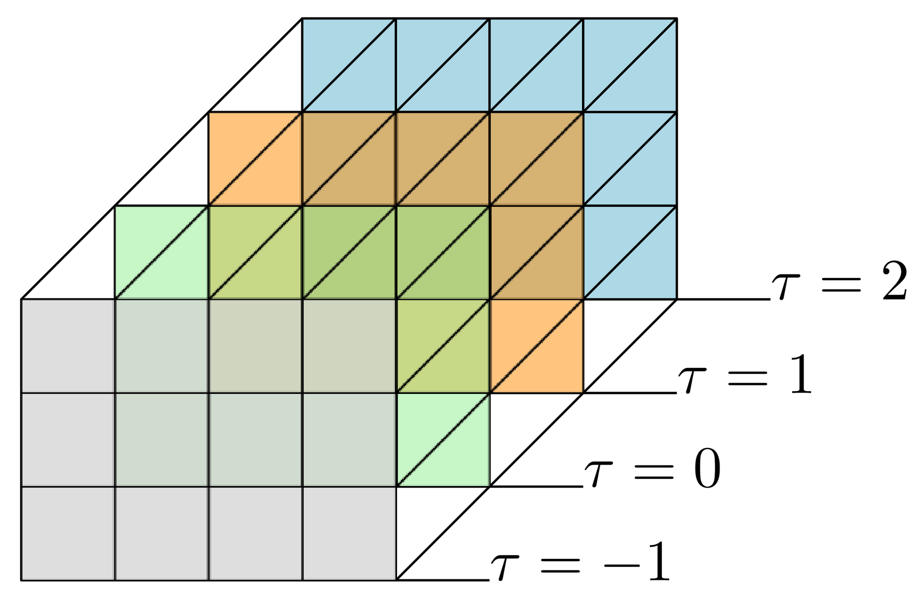
Proposed Algorithm

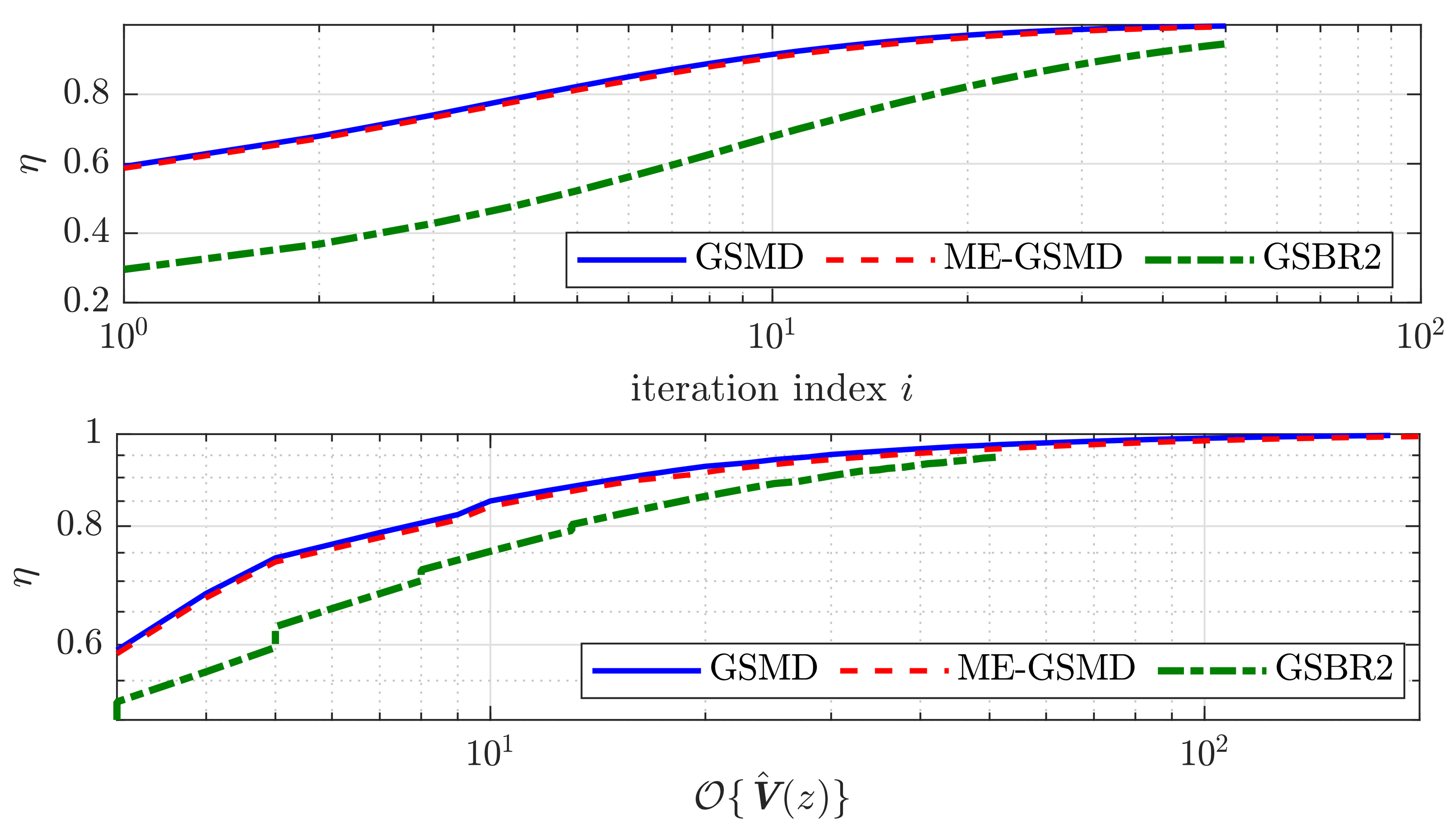
Assuming $\mathbf{A}[\tau] \in \mathbb{C}^{M \times L}$ denotes time-domain equivalent of $\boldsymbol{A}(z)$ where $\tau \in \mathbb{Z}$ can be both positive and negative integers with zero-lag slice at $\tau = 0$. Example 3×4 polynomial matrix



Procedure involves repetition of the following steps:

1. perform search for maximum off-diagonal element or column with maximum norm excluding off-diagonal term.
2. time shift the element or the column onto zero-lag via paraunitary time-shift.
3. compute conventional SVD of zero-lag and applies its unitary matrices to all lags.

Simulation & Results



- Both GSMD and ME-GSMD converge faster than GSBR2.
- It produces lower-order paraunitary matrices $\hat{\boldsymbol{V}}(z)$ and $\hat{\boldsymbol{U}}(z)$
- Unlike GSBR2, variety of variants are possible i.e. multiple shift, divide-and-conquer, extra-shifts

$\eta = \frac{\sum_\tau ||\bar{\boldsymbol{\Sigma}}[\tau]||_{\mathrm{F}}^2}{\sum_\tau ||\hat{\boldsymbol{\Sigma}}[\tau]||_{\mathrm{F}}^2}$, $\bar{\hat{\boldsymbol{\Sigma}}}[\tau]$ is same as $\boldsymbol{\Sigma}[\tau]$ with off-diagonal set to zero.
ME-GSMD uses the ∞-norm, while GSMD uses the 2-norm in maximum search.