Polynomial Subspace Decomposition for Broadband Angle of Arrival Estimation

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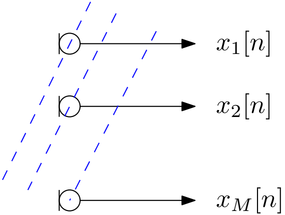
Presentation Overview

1. Broadband array setup and covariance matrix
2. (Narrowband) angle of arrival estimation
3. Polynomial eigenvalue decomposition (PEVD) and subspaces
4. (Broadband) polynomial MUSIC algorithms
5. Iterative PEVD algorithms:
   - second order sequential best rotation (SBR2)
   - sequential matrix diagonalisation (MSME-SMD)
6. Simulations & Results
7. Conclusions

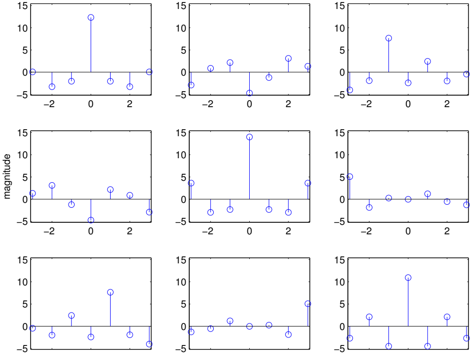
Broadband Array Setup and Covariance



- space-time covariance matrix:

$$\mathbf{R}[\tau] = \mathcal{E}\{\mathbf{x}_n \mathbf{x}_{n-\tau}^{\mathrm{H}}\}$$

- a matrix of auto- and cross-correlation sequences
- symmetry: $\mathbf{R}[\tau] = \mathbf{R}^{\mathrm{H}}[-\tau]$



- cross-spectral density

$$\boldsymbol{R}(z) = \sum_{-T}^{T} \mathbf{R}[\tau] z^{-\tau}$$

is a polynomial matrix

- parahermitian:

$$\tilde{\boldsymbol{R}}(z) = \boldsymbol{R}^{\mathrm{H}}(z^{-1}) = \boldsymbol{R}(z)$$

Broadband Steering Vector

- A signal $s[n]$ arriving at the array can be characterised by the delays of its wavefront (neglecting attenuation):

$$\begin{bmatrix} x_0[n] \\ x_1[n] \\ \vdots \\ x_{M-1}[n] \end{bmatrix} = \begin{bmatrix} s[n-\tau_0] \\ s[n-\tau_1] \\ \vdots \\ s[n-\tau_{M-1}] \end{bmatrix} = \begin{bmatrix} \delta[n-\tau_0] \\ \delta[n-\tau_1] \\ \vdots \\ \delta[n-\tau_{M-1}] \end{bmatrix} * s[n] \circ\!\!-\!\!\bullet\ \mathbf{a}_\vartheta(z)S(z)$$

- signal model in the case of a mixture of source signals:

$$\mathbf{R}[\tau] = \mathcal{E}\{\mathbf{x}_n \mathbf{x}_{n-\tau}^{\mathrm{H}}\} \quad \circ\!\!-\!\!\bullet \quad \boldsymbol{R}(z) = \sum_l S_l(z)\mathbf{a}_{\vartheta_l}(z)\tilde{\mathbf{a}}_{\vartheta_l}(z) + \sigma_N^2 \mathbf{I}$$

  with ϑ_l the direction of arrival and $S_l(z)$ the PSD of the lth source;

- narrowband covariance matrix only contains instantaneous correlations $\mathbf{R} = \mathbf{R}[0]$.

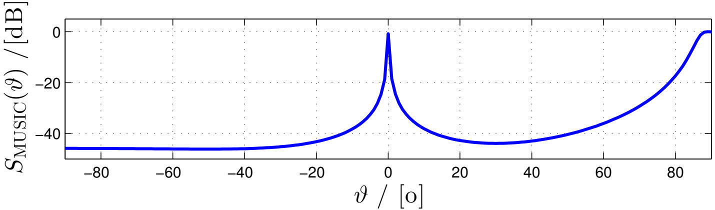
Narrowband MUSIC Algorithm

- EVD of the narrowband covariance matrix $\mathbf{R}$ identifies signal-plus-noise and noise-only subspaces

$$\mathbf{R} = [\mathbf{U}_s \;\; \mathbf{U}_n] \begin{bmatrix} \mathbf{\Lambda}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_n \end{bmatrix} \begin{bmatrix} \mathbf{U}_s^{\mathrm{H}} \\ \mathbf{U}_n^{\mathrm{H}} \end{bmatrix}$$

- scanning the signal-plus-noise subspace could only help to retrieve sources with orthogonal steering vectors;
- therefore, the multiple signal classification (MUSIC) algorithm scans the noise-only subspace for minima, or maxima of its reciprocal

$$S_{\mathrm{MUSIC}}(\vartheta) = \frac{1}{\|\mathbf{U}_n \mathbf{a}_{\vartheta,\Omega_i}\|_2^2}$$



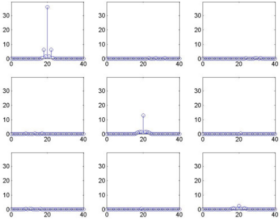
Polynomial Eigenvalue Decomposition

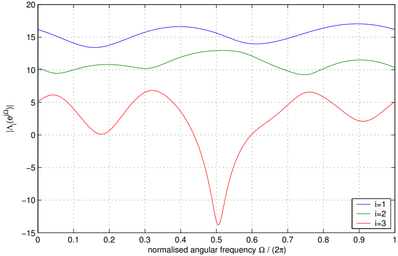
[McWhirter *et al.*,*IEEE Trans SP* (2007)]

- Based on the polynomial EVD of the broadband covariance matrix

$$\mathbf{R}(z) \approx \underbrace{\left[\mathbf{Q}_s(z) \quad \mathbf{Q}_n(z)\right]}_{\mathbf{Q}(z)} \underbrace{\begin{bmatrix} \mathbf{\Lambda}_s(z) & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_n(z) \end{bmatrix}}_{\mathbf{\Lambda}(z)} \begin{bmatrix} \tilde{\mathbf{Q}}_s(z) \\ \tilde{\mathbf{Q}}_n(z) \end{bmatrix}$$

- paraunitary $\mathbf{Q}(z)$, s.t. $\mathbf{Q}(z)\tilde{\mathbf{Q}}(z) = \mathbf{I}$;
- diagonalised and spectrally majorised $\mathbf{\Lambda}(z)$:





Polynomial MUSIC Algorithms

[Alrmah *et al.*,*EUSIPCO* (2011)]

Polynomial Spatial (PS) MUSIC Algorithm

- similar to MUSIC, scan the null space by broadband steering vectors $\mathbf{a}_{\vartheta}(z)$;
- minimum energy of $\tilde{\mathbf{Q}}_n(z)\mathbf{a}_{\vartheta}(z)$ is indicative of a source at AoA $\Rightarrow (\vartheta)$

$$\Gamma_{\vartheta}(z) = \tilde{\mathbf{a}}_{\vartheta}(z)\mathbf{Q}_n(z)\tilde{\mathbf{Q}}_n(z)\mathbf{a}_{\vartheta}(z)$$

- Polynomial spatial MUSIC spectrum:

$$S_{\text{PS-MUSIC}}(\vartheta) = 1/\gamma_{\vartheta}[0]$$

Polynomial Spatio-Spectral (PSS) MUSIC Algorithm

- using the spectral information contained in the power spectral density term $\Rightarrow \Gamma_{\vartheta}(z)$

$$S_{\text{PSS-MUSIC}}(\vartheta, \Omega) = \Gamma_{\vartheta}^{-1}(e^{j\Omega})$$

- this metric can also determine over which frequency range sources in the direction defined by the steering vector $\mathbf{a}_{\vartheta}(z)$ are active.

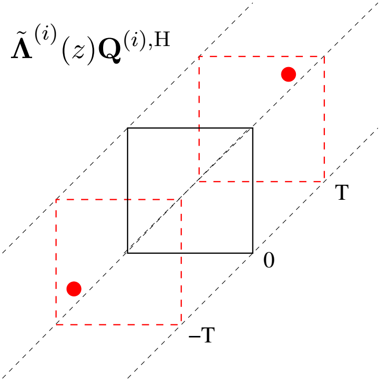
Second Order Sequential Best Rotation (SBR2)

[McWhirter *et al.*, *IEEE Trans SP* (2007)]

- SBR2 is an iterative method to approximate the PEVD, and can be seen as a generalised Jacobi algorithm;
- at each iteration, an elementary paraunitary operation transfers the largest off-diagonal element onto the main diagonal;
- with $\boldsymbol{S}^{(0)}(z) = \boldsymbol{R}(z)$, in the ith iteration:

$$\boldsymbol{S}^{(i)}(z) = \mathbf{Q}^{(i)}\boldsymbol{\Lambda}^{(i)}(z)\ \ \boldsymbol{S}^{(i-1)}(z)\ \ \tilde{\boldsymbol{\Lambda}}^{(i)}(z)\mathbf{Q}^{(i),\mathrm{H}}$$

- visualisation of maximum off-diagonal elements:
- stop once off-diagonal elements fall below a defined threshold
- SBR2 is guaranteed to converge



Sequential Matrix Diagonalisation (SMD)

[Redif *et al.*, submitted to *IEEE Trans SP* (2014)]

- SMD operates in a similar iterative fashion to SBR2, using a sequence of paraunitary operations to approximate the PEVD;
- at each iteration, the zero-lag matrix is diagonalised by an EVD;
- a unitary matrix is applied to matrix-coefficients at all lags.

Multiple-Shift SMD (MS-SMD)

[Corr *et al.*, *SSP* (2014)]

- By multiple time-shift operations, the zero-lag matrix is filled as best as possible with large coefficients;
- similar complexity to SMD, but faster convergence.

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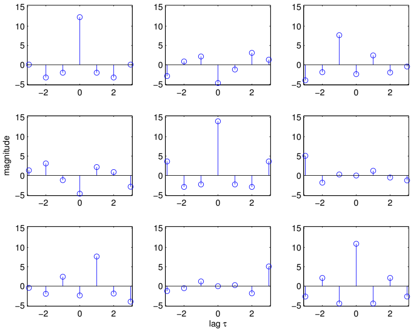
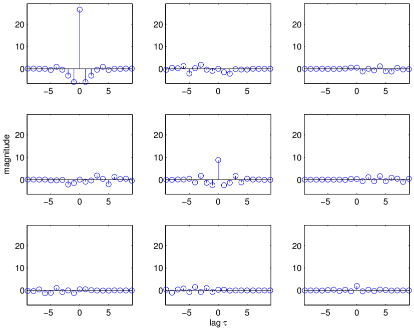
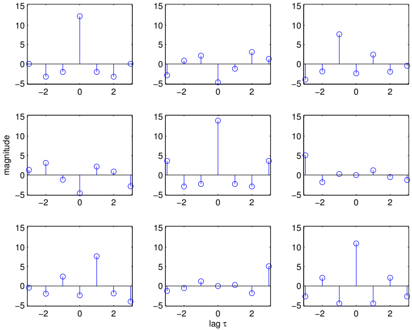
Results - Before & After PEVD

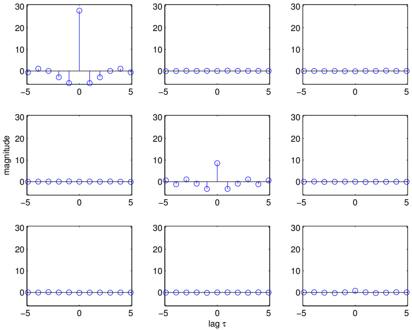




- original $\mathbf{R}(z)$
- $\boldsymbol{S}^{(5)}(z)$ after 5 iterations
- performance metric for diagonalisation: remaining off-diagonal energy

Results - Before & After PEVD

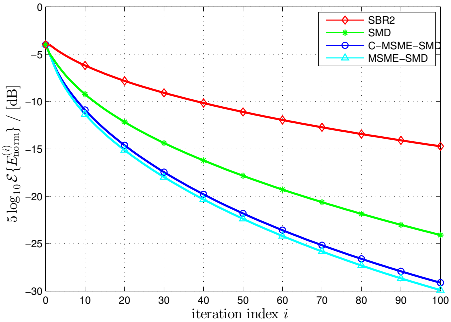


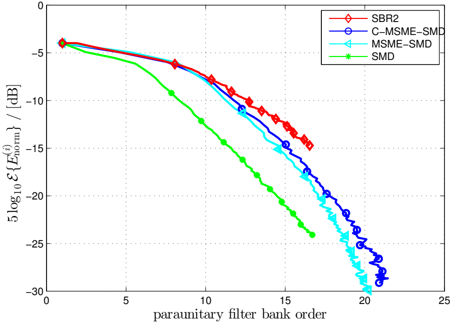


- original $\mathbf{R}(z)$
- $\boldsymbol{S}^{(50)}(z)$ after 50 iterations
- performance metric for diagonalisation: remaining off-diagonal energy

SBR2 / SMD / MS-SMD Comparison

- Remaining off-diagonal energy as a metrix; results averaged over an ensemble 10^3 of random 5x5x11 parahermitian matrices

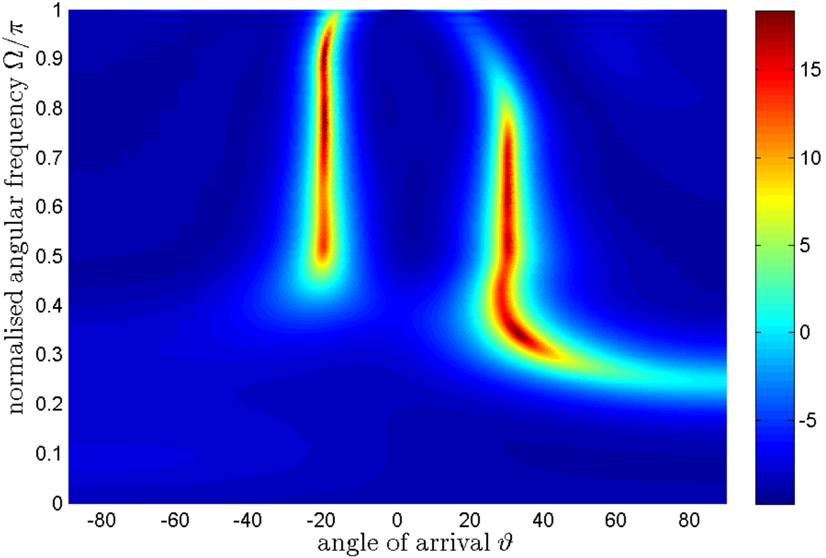




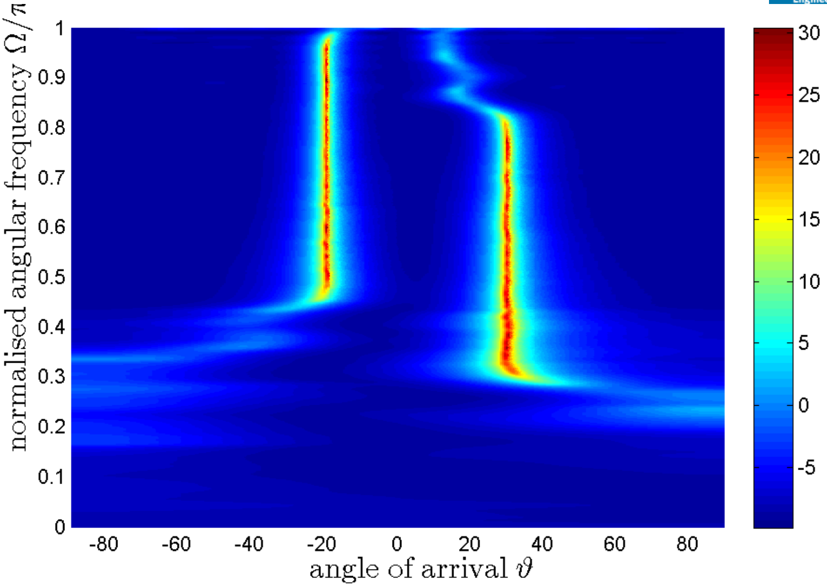
Simulation Setup

- Linear uniform array with $M = 8$ elements;
- scenario with two sources:
  1. source one at $\vartheta_1 = 30^\circ$, active within $\Omega_1 \in \{0.3125\pi; 0.7812\pi\}$
  2. source two at $\vartheta_2 = -20^\circ$, active within $\Omega_2 \in \{0.4688\pi; 0.9375\pi\}$
- corrupted by white Gaussian noise at 20dB SNR.

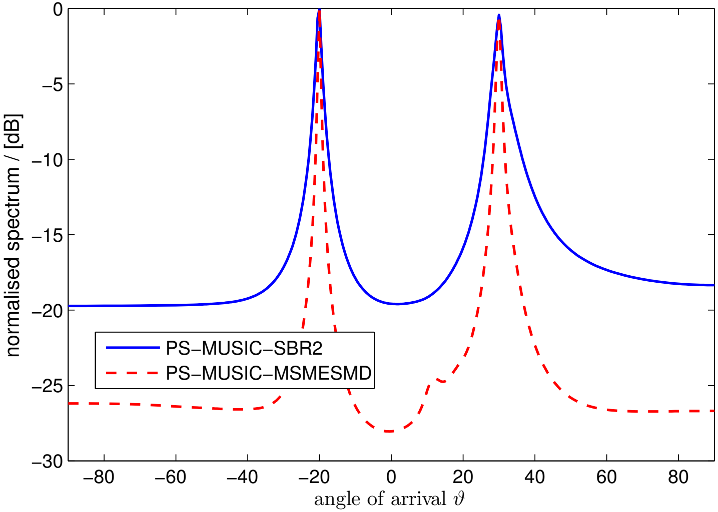
PSS-MUSIC with SBR2



PSS-MUSIC with MS-SMD



PS-MUSIC Comparison



Conclusions

- The need for a polynomial EVD has been motivated using broadband array processing;
- several iterative PEVD algorithms exist: SBR2, SMD and various versions thereof;
- recent iterative PEVD algorithms show excellent diagonalisation;
- this translates into more accurate post-processing when relying on the PEVD;
- as an example, we have provided results for broadband angle of arrival estimation using a polynomial MUSIC method.