**Introduction:** The aim of this work package is to understand and model difficult and complex environments. Traditional algorithms for detection, classification or identification are based on simplistic models of noise, clutter or multipath. Therefore most of them fail to achieve useful or meaningful results. We aim to develop realistic physical-based models for the full sensing chain from the sensors themselves to the complex interaction with clutter/target and propagation into the environment. A physical understanding of the clutter rather than ad hoc and simple models will help to develop new DLC (Detection, Localisation and Classification) algorithms with optimal performances with reduced computational power as well as in situ environment adaptability for greater robustness.

**Scope and interaction with other WPs**

WP3.1: Build DLC framework for estimation of time-varying number of objects in complex cluttered scenarios
- Design complex clutter models (time + space correlation);
- Integrate clutter to novel multi-object estimation techniques to build a unified DLC framework;
- Exploit parallelization of computational resources for algorithm design (WP6.1).

WP3.2: Enrich DLC framework with target physical modelling
- Study and design appropriate physical models for target signatures;
- Integrate physical modelling to unified DLC framework;
- Combine unified framework with Distributed Decentralised Detection to address IED threat (WP2.2).

WP3.3: Extend DLC framework to address man-made object detection
- Shape clutter model to accommodate for man-made object detection;
- Extend DLC techniques to adapt signals to local environment (WP5.1).

One of the strengths of the UDRC program is the collaboration between the WPs. With distinct expertise collaborations will lead to fruitful results and enable us to tackle more complex problems.

- WP2: Development of realistic models for underwater sensors (sonobuoys) and for underwater acoustic communication.
- WP5: Automatic tracking using MIMO sonar systems
- WP6: Development of efficient algorithms for wave propagation, interaction between sound and objects.

**Sonar MIMO formulation**

Haimovich proposed a formulation for narrowband MIMO radar. For a MIMO radar system with \( K \) transmitters and \( L \) receivers, the signal \( z_{ik}(t) \) received by the receiver \( k \) from the transmitter \( i \) and interacting with a point target \((X_q)_{q=1}^{Q} \) can be written as:

\[
\begin{aligned}
z_{ik}(t) &= \sqrt{\frac{P}{K}} \sum_{q=1}^{Q} h_{ik}^{q} \left( t - \tau_{ik}(X_q) - \tau_{qi}(X_q) \right) \\
&= \sqrt{\frac{P}{K}} \sum_{q=1}^{Q} h_{ik}^{q} \left( t - \tau_{ik}(X_q) - \tau_{qi}(X_q) \right)
\end{aligned}
\]

(1)

where \( h_{ik}^{q} \) is the propagation time delay between the transmitter \( i \) and the scattering point \( X_q \). The target response \( \tau_{ik}(X_q) \) represents the propagation delay between the transmitter \( i \) and the scattering point \( X_q \) and the receiver \( k \).

For broadband MIMO sonar systems we can write in the Fourier domain:

\[
Z_{ik}(\omega) = \sqrt{\frac{P}{K}} \sum_{q=1}^{Q} h_{ik}^{q} \mathcal{F}_2(\omega) e^{-j\omega\tau_{ik}(X_q;\gamma_i(X_q;\gamma_q))}
\]

(3)

where \( \mathcal{F}_2(\omega) \) is the Fourier transform of the transmitted signal from the transmitter and \( \mathcal{F}_2(\omega) \) is the Fourier transform of the transmitted signal from the receiver.

\[
Z_{ik}(\omega) = \mathcal{F}_2(\omega) H_{ik}(\omega) \mathcal{F}_2(\omega) e^{-j\omega\tau_{ik}(X_q;\gamma_i(X_q;\gamma_q))}
\]

(4)

where \( \gamma_i \) is the angle of view of the target from the transmitter and \( \gamma_q \) is the angle of view of the target from the receiver. Eq. (4) can be interpreted as follows: the first term corresponds to the propagation of the wave to and from the target, the second term is the form function of the target, the third term is the transmitted signal. This formulation can be easily generalised to include multipath and attenuation terms.

\[
Z_{ik}(\omega) = \sum_{p=1}^{P} (\omega) H_{ik}(\omega) \mathcal{F}_2(\omega) e^{-j\omega\tau_{ik}(X_q;\gamma_i(X_q;\gamma_q))}
\]

(5)

**MIMO identification capability**

The target response \( \gamma_{ik}(\omega) \) in Eq. (1) corresponds to the essence of a random walk in the complex plane where each step is modelled by a random variable \( 1/\sqrt{2\pi}e^{it\gamma} \) where \( U \in [0,1] \) is the uniform distribution. Thanks to the central limit theorem we can compute the limit:

\[
\lim_{Q \to \infty} \frac{1}{\sqrt{Q}} \sum_{q=1}^{Q} h_{ik}^{q} = \text{Rayleigh}(1/\sqrt{2})
\]

(6)

The probability density functions of targets with a small number of scatterers (\( \leq 5 \)) however are very distinguishable. By using Bayes rule we can compute the probability that a target has \( q \) scatterers given a set of observations \( \Gamma \):

\[
P(q|\Gamma) = \frac{\prod_{Q=1}^{Q} P(q|\Gamma)}{\sum_{Q=1}^{Q} P(q|\Gamma)}
\]

(7)

**Super resolution using MIMO systems**

Let \( r \) be the MIMO response of the target \((X_q)\), the detection rule function \( F(r) \) can be written as:

\[
F(r) = \frac{1}{N} \sum_{k=1}^{N} \sum_{l=1}^{N} R(|r_{ik} - r_{lk}|) \sim \frac{1}{N} \text{Rayleigh}^2(r)
\]

(8)

We can prove that:

\[
\lim_{N \to \infty} \text{Rayleigh}^2(r)
\]

(9)

So large MIMO systems decorrelate individual scatterers within a cell resolution and then achieve super resolution.

**Conclusion:** The MIMO sonar capabilities described here make such a system a very attractive tool for surveillance. In a fixed environment such as an harbour or a narrow channel the transmitters and receivers elements can be carefully placed to ensure coverage and view independence. The recognition capabilities of MIMO sonar can then be used to identify threats. All these features were studied using a single snapshot of MIMO systems. In future work and in conjunction with WP5 we will study the dynamic aspect of sonar MIMO systems by introducing an automatic target tracking component.

This work was supported by the Engineering and Physical Sciences Research Council (EPSRC) and Dstl.