Loughborough University

Transmit Beamforming Design for Two-Dimensional Phased-MIMO Radar with Fully-Overlapped Subarrays

A. Deligiannis, S. Lambotharan, J.A. Chambers
School of Electronic, Electrical and Systems Engineering
Loughborough University

SSPD, Edinburgh

2014, 8-9 September

Outline

1. Introduction
2. Fully-Overlapped System Model
3. Transmit Beamforming Design
4. Simulation Results
5. Conclusion

Loughborough University

Outline

1. Introduction
2. Fully-Overlapped System Model
3. Transmit Beamforming Design
4. Simulation Results
5. Conclusion

Loughborough University

Introduction

- We investigate a subaperturing technique for two-dimensional (2D) transmit arrays within the context of MIMO radar.
- We examine the performance of transmit beamforming using fully-overlapped subarrays of a 2D transmit array.
- As reported for linear array of antennas, this technique exploits the advantages of both MIMO radar technology and phased-array radar.
- The purpose is to focus the energy of the transmit array into a desired 2D spatial sector, while constraining the sidelobe levels to low values.
- Our aim is achieved by solving a convex optimization problem that minimizes the difference between a desired transmit beampattern and the actual beampattern produced by our system.

Loughborough University

MIMO Radar

MIMO radar is characterized by employing multiple antennas to simultaneously transmit probing signals that may be chosen to be either correlated or uncorrelated and by using multiple antennas to receive the reflected signals.

Widely separated antennas:
- Ability to capture the spatial diversity of the target's radar cross section (RCS)
- Improvement of target detection performance
- Accurate parameter estimation of rapidly moving targets

Colocated antennas:
- Higher resolution
- Higher sensitivity to detect slowly moving targets
- Better parameter identifiability
- Direct applicability of adaptive array techniques

MIMO Radar

The aforementioned advantages offered by MIMO radar technology come at the cost of losing the transmit coherent processing gain offered by the phased-array radar. This absence leads to:

- Beam shape loss
- Performance degradation in the presence of the target's RCS
- Signal-to-noise ratio (SNR) gain decrease

To overpower these weaknesses Hassanien and Vorobyov* developed a new radar technique, known as Phased-MIMO radar. The innovative idea of partitioning the transmit array into a number of subarrays that are allowed to overlap overcomes the loss of transmit coherent gain and jointly exploits the benefits of the phased-array and MIMO radars. Furthermore, Phased-MIMO radar offers a tradeoff between angular resolution and robustness against beam-shape loss by properly selecting the number of subarrays.

*: A. Hassanien, and S. A. Vorobyov, "Phased-MIMO Radar: A Tradeoff Between Phased-Array and MIMO Radars". IEEE Trans. Signal Processing, **58**(6): 3137-3151, 2010.

Loughborough University

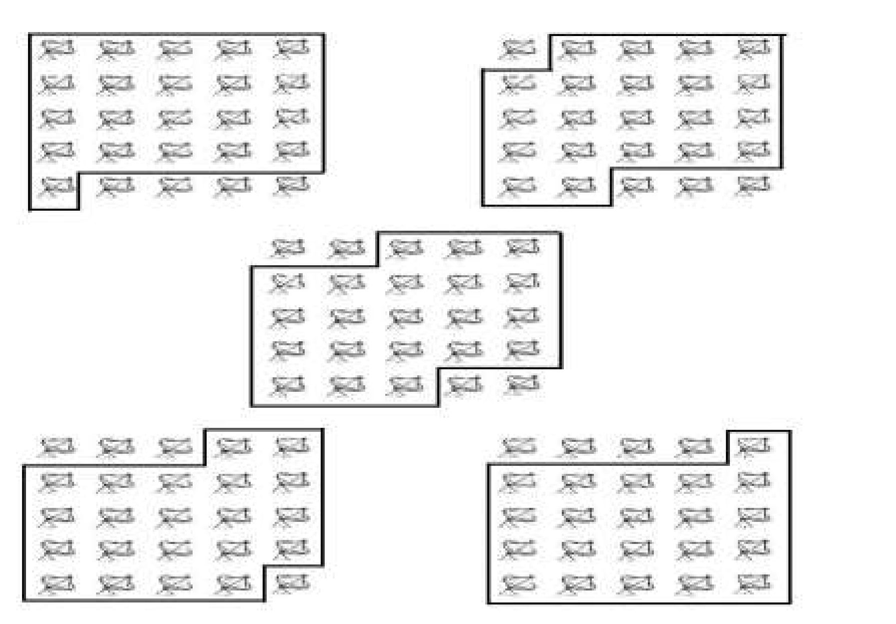
Outline

1. Introduction
2. Fully-Overlapped System Model
3. Transmit Beamforming Design
4. Simulation Results
5. Conclusion

Loughborough University

System Model

- A radar system that incorporates a uniform rectangular array (URA) at the transmit side, which consists of $M_t \times N_t$ antennas.
- The main idea is to partition the 2D transmit array into K subarrays ($1 \leq K \leq M_t \times N_t$), which are fully overlapped.
- Our aim is to focus the energy of the transmit array into a 2D spatial sector defined by Θ= $[\theta 1 \ \ \theta 2]$ in the elevation domain and Φ= $[\varphi 1 \ \ \varphi 2]$ in the azimuth domain.
- We form K transmit beams, each of them is steered by the corresponding subarray.



System Model

- The power of the probing signal emitted by the k^{th} subarray can be modeled as:

$$P_k(\theta,\varphi) = \boldsymbol{a}_k^H(\theta,\varphi)\mathrm{E}\{\boldsymbol{s}_k(t)\boldsymbol{s}_{\boldsymbol{k}}^H(t)\}\,\boldsymbol{a}_k(\theta,\varphi)$$
$$=\boldsymbol{a}_k^H(\theta,\varphi)\boldsymbol{w}_k\boldsymbol{w}_k^H\boldsymbol{a}_k(\theta,\varphi)$$

where:

- $\boldsymbol{a}_k(\theta,\varphi)$ is the steering vector associated with the k^{th} subarray
- $\boldsymbol{s}_k(t)$ is the complex envelope of the signals at the output of the k^{th} subarray and can be designed by $\boldsymbol{s}_k(t) = \boldsymbol{w}_k\psi_k(t)$
- $\boldsymbol{w}_k \in \ C^{M_tN_t \times 1}$ is the transmit weight vector, used to form the k^{th} transmit beam
- $\psi_k(t)$ is the independent waveform vector of size $K\ \times 1$
- t refers to the time index within the radar pulse

- The total transmit power defines the array transmit beampattern:

$$P_k(\theta,\varphi) = \sum_{k=1}^{K} \boldsymbol{a}_k^H(\theta,\varphi)\boldsymbol{w}_k\boldsymbol{w}_k^H\boldsymbol{a}_k(\theta,\varphi)$$

Outline

1. Introduction
2. Fully-Overlapped System Model
3. Transmit Beamforming Design
4. Simulation Results
5. Conclusion

Transmit Beamforming Design

- We derive the optimization problem of minimizing the maximum difference between the desired 2D transmit beampattern and the transmit beampattern of our system:

$$\min_{\boldsymbol{w}_1,\ldots,\boldsymbol{w}_K} \quad \max_{\theta,\varphi} \left| P_d(\theta,\varphi) - \sum_{k=1}^{K} \boldsymbol{w}_k^H \boldsymbol{a}_k(\theta,\varphi) \boldsymbol{a}_k^H(\theta,\varphi) \boldsymbol{w}_k \right|$$

$$s.t. \quad \sum_{k=1}^{K} |\boldsymbol{W}_{[lk]}|^2 = \frac{E}{M_t N_t - (K-1)}, \; l = 1, \ldots, M_t N_t$$

where

- $\boldsymbol{W} = [\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K] \in C^{M_t N_t \times K}$ is the transmit beampattern weight matrix
- $P_d(\theta,\varphi)$ is the desired beampattern
- E is the total available power

This optimization problem is a non-convex quadratically constrained quadratic programming (QCQP) problem, which is NP-hard to solve!

Transmit Beamforming Design

- In order to recast our problem as a convex one, we use the semidefinite relaxation technique, by defining a matrix $\boldsymbol{X_k} = \boldsymbol{w_k}\boldsymbol{w_k^H} \in C^{M_tN_t \times M_tN_t}$, $k = 1, \ldots, K$ and reformulating our problem as:

$$\min_{\boldsymbol{w}_1,\ldots,\boldsymbol{w}_K} \quad \max_{\theta,\varphi} \left| P_d(\theta,\varphi) - \sum_{k=1}^{K} Tr\{\boldsymbol{a}_k(\theta,\varphi)\boldsymbol{a}_k^{\boldsymbol{H}}(\theta,\varphi)\, \boldsymbol{X}_k\} \right|$$

$$s.t. \sum_{k=1}^{K} diag\{\boldsymbol{X}_k\} = \frac{E}{M_tN_t - (K-1)} \mathbf{1}_{M_tN_t \times 1}$$

$$\boldsymbol{X}_k \geq 0, \quad k = 1, \ldots, K$$

$$rank(\boldsymbol{X}_k) = 1, \quad \mathrm{k} = 1, \ldots, \mathrm{K}$$

- The rank constraint maintains the optimization problem as non-convex.
- Relaxing the rank constraint we recast the problem as a convex one and solve it using semidefinite programming (SDP).
- The next step is to extract the transmit weight vectors $\boldsymbol{w_k}$ from the optimal solution denoted as $\boldsymbol{X}_k^*$ for $k = 1, \ldots, K$.
- If the rank of $\boldsymbol{X}_k^*$ is one, the optimal $\boldsymbol{w_k}$ is obtained straightforwardly as the eigenvector of $\boldsymbol{X}_k^*$, corresponding to the principal eigenvalue. In the case of $rank(\boldsymbol{X}_k^*) > 1$, we use randomization techniques to derive the optimal $\boldsymbol{w_k}$.

Outline

1. Introduction
2. Fully-Overlapped System Model
3. Transmit Beamforming Design
4. Simulation Results
5. Conclusion

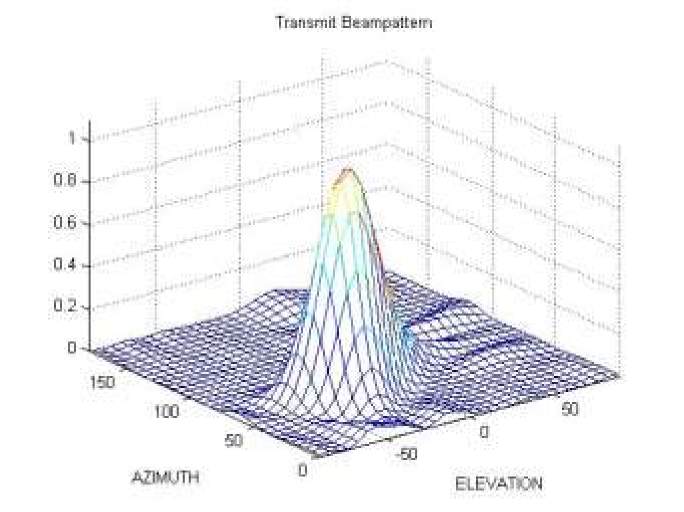
Loughborough University

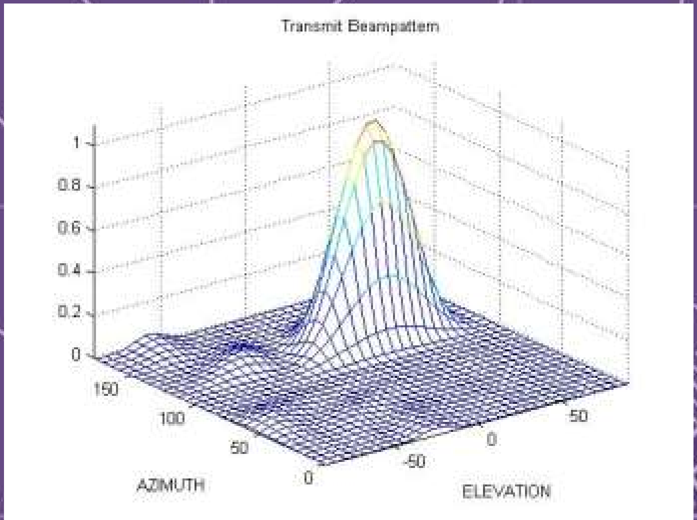
Simulation Results

- We assume a 5×5 transmit URA with half-wavelength spacing between adjacent antennas.

Example 1:The transmit array is divided into 5 subarrays . The desired beampattern has a mainlobe defined by the 2D sector $\Theta = [-40^{\circ}, -20^{\circ}]$ in the elevation domain and $\Phi = [50^{\circ}, 85^{\circ}]$ in the azimuth domain.

Example 2:The transmit array is divided into 7 subarrays . The desired beampattern has a mainlobe defined by the 2D sector $\Theta = [15^{\circ}, 55^{\circ}]$ in the elevation domain and $\Phi = [110^{\circ}, 140^{\circ}]$ in the azimuth domain.



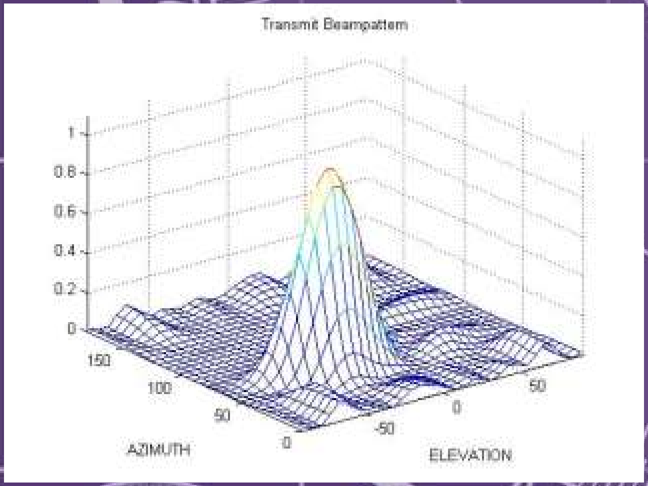


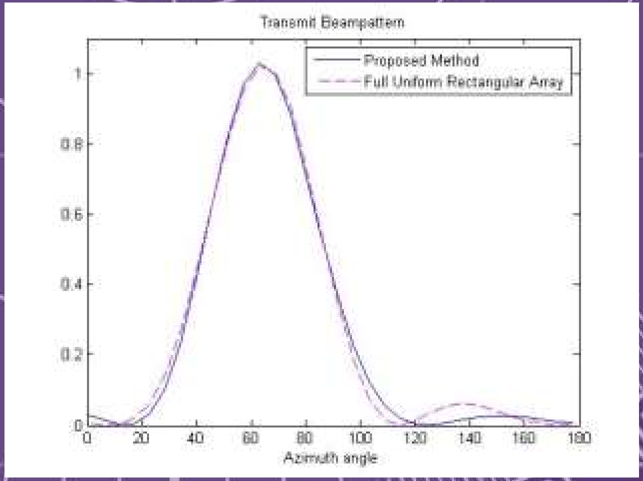
Loughborough University

Simulation Results

Example 3: Comparison of the proposed subaperturing technique with the case when the URA uses all of its elements when transmitting the probing signal. The 2D sector of interest is defined as in the first example in order to facilitate the comparison ($\Theta = [-40^{\circ}, -20^{\circ}]$ and $\Phi = [50^{\circ}, 85^{\circ}]$). We use 5 transmit beams to synthesize the 2D transmit beampattern.

Example 4: Comparison of the cross section of the transmit beampattern, plotted against the azimuth angle by keeping the elevation angle constant at -27°.





Outline

1. Introduction
2. Fully-Overlapped System Model
3. Transmit Beamforming Design
4. Simulation Results
5. Conclusion

Conclusion

- We investigated the problem of 2D transmit beamforming design for the MIMO radar with fully overlapped subarrays.
- The simulation results confirm that the system transmit beampattern approximates the desired sector of space with high accuracy.
- The sidelobe levels are very low and are restricted in an area close to the mainlobe, without covering the whole 2D space.
- Finally, the comparison between the proposed method and the full URA case proves that the concentration of the power within the desired 2D sector is more evident in the proposed method.

Loughborough University

Thank you!
Any questions?

Loughborough University